

NRF 10: Chapter 3 Review

Factoring and Expanding.

Answers are in RED.

| Expanding (Multiplying) | Factoring (Dividing) |
|---|--|
| <p style="text-align: center;">Type 1: Monomial x Binomial or Monomial x Trinomial</p> <p>Ex: $3(x + 4) = 3x + 12$ $-2(x^2 + 2x - 1) = -2x^2 - 4x + 2$</p> <p><u>Questions to try:</u></p> <ol style="list-style-type: none"> 1. $5(x - 3)$ $5x - 15$ 2. $x(x + 5)$ $x^2 + 5x$ 3. $2x(x^2 - 3x + 2)$ $2x^3 - 6x^2 + 4x$ 4. $-4(x - 5)$ $-4x + 20$ 5. $-3x(x + 2y)$ $-3x^2 - 6xy$ | <p style="text-align: center;">Type 1: Factor with a GCF</p> <p>Ex: $2x + 10$, GCF is 2 : $2(x + 5)$ $-3x^2 + 6x$, GCF is $-3x$: $-3x(x - 2)$</p> <p><u>Questions to try:</u></p> <ol style="list-style-type: none"> 1. $4x + 12$ $4(x + 3)$ 2. $x^2 - 6xy$ $x(x - 6y)$ 3. $10x^2 + 20x$ $10x(x + 2)$ 4. $-5x - 10$ $-5(x + 2)$ 5. $-4x^2y + 8xy$ $-4xy(x - 2)$ |
| <p>Always check to see if your polynomial can be <u>simplified by finding a GCF</u> BEFORE using one of the following methods of FACTORING!!!!</p> | |
| <p style="text-align: center;">Type 2: Binomial x Binomial</p> <p>Use distributive property: Multiply each term in the 1st bracket with everything in the 2nd bracket and then combine like terms.</p> <p>Ex: $(x + 3)(x + 4)$ $x(x + 4) + 3(x + 4)$ $x^2 + 4x + 3x + 12$ $x^2 + 7x + 12$</p> <p>$(x - 3)(x + 5)$ $x^2 + 5x - 3x - 15$ $x^2 + 2x - 15$</p> <p><u>Questions to try:</u></p> <ol style="list-style-type: none"> 1. $(x + 2)(x + 5)$ $x^2 + 7x + 10$ 2. $(y + 1)(y + 7)$ $y^2 + 8y + 7$ 3. $(x - 5)(x - 7)$ $x^2 - 12x + 35$ 4. $(y - 2)(y - 3)$ $y^2 - 5y + 6$ 5. $(x + 6)(x - 8)$ $x^2 - 2x - 48$ | <p style="text-align: center;">Type 2: Factoring Trinomials, form $x^2 + bx + c$</p> <p>Find 2 numbers that multiply to give us the last number, the "c" and the same 2 numbers will add to give us the middle number or the coefficient on the x-term, the "b".</p> <p>Ex: $x^2 + 8x + 15$ 3 and 5 multiply to make 15 and add to make 8, so: $(x + 3)(x + 5)$</p> <p>Or $x^2 - 2x - 15 = (x - 5)(x + 3)$</p> <p><u>Questions to try:</u></p> <ol style="list-style-type: none"> 1. $x^2 + 10x + 16$ $(x + 8)(x + 2)$ 2. $y^2 + 9y + 18$ $(y + 6)(y + 3)$ 3. $x^2 - 7x - 18$ $(x - 9)(x + 2)$ 4. $y^2 + 2y - 63$ $(y + 9)(y - 7)$ 5. $y^2 + 5y - 36$ $(y + 9)(y - 4)$ |

Type 3: Binomial x Binomial

Use distributive property: Multiply each term in the 1st bracket with everything in the 2nd bracket and then combine like terms.

Ex: $(2x + 3)(3x + 4)$
 $2x(3x + 4) + 3(3x + 4)$
 $6x^2 + 8x + 9x + 12$
 $6x^2 + 17x + 12$

Coefficients on the x's here!!

Questions to try:

- $(2x + 5)(x + 3)$ $2x^2 + 11x + 15$
- $(3x + 2)(2x + 1)$ $6x^2 + 7x + 2$
- $(x - 4)(3x - 5)$ $3x^2 - 17x + 20$
- $(2x + 1)(3x - 2)$ $6x^2 - x - 2$

Type 3: Factoring Trinomials, form $ax^2 + bx + c$, factoring by DECOMPOSITION

There is a coefficient on the x^2 -term.

Ex: $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

- Multiply the coefficient on the x^2 -term with the constant term ($2 \times 2 = 4$)
- Find 2 numbers that will multiply to give answer from 1st step and will add to give the coefficient on x-term. So, in this case **1 and 4** will multiply to give 4 and add to give 5.
- Re-write the polynomial by breaking the x-term into 2 terms, using the numbers that we just found in previous step
 $2x^2 + 4x + 1x + 2$
- Break the newly written polynomial into 2 binomials and find GCF for each binomial.
 $2x^2 + 4x$: GCF is $2x$, so $2x(x + 2)$
 $1x + 2$: GCF is 1 , so $1(x + 2)$
- Write as a product of factors:
 $(2x + 1)(x + 2)$

Questions to try:

- $2x^2 + 13x + 15$ $(2x + 3)(x + 5)$
- $6x^2 + 11x + 3$ $(2x + 3)(3x + 1)$
- $2x^2 - 3x - 9$ $(2x + 3)(x - 3)$
- $3x^2 - 19x + 20$ $(3x - 4)(x - 5)$

Type 4: Binomial²

Or the same binomial is multiplied by itself.

Ex: $(3x - 2)^2$
 $(3x - 2)(3x - 2)$
 $9x^2 - 6x - 6x + 4$
 $9x^2 - 12x + 4$

Questions to try:

- $(4x - 2)^2$ $16x^2 - 16x + 4$
- $(2x + 3)^2$ $4x^2 + 12x + 9$
- $(5x - 2)^2$ $25x^2 - 20x + 4$
- $(3x + 6)^2$ $9x^2 + 36x + 36$

Type 4: Factoring a Perfect Square Trinomial

Look for a trinomial whose x^2 coefficient and constant term are perfect squares.

Ex: $49x^2 + 56x + 16$
 $\sqrt{49x^2} = 7x$ and $\sqrt{16} = 4$, so:
 $(7x + 4)(7x + 4)$
 $(7x + 4)^2$

Be sure to check that the polynomial that you factored was a Perfect Square Trinomial by multiplying the 2 terms from your answer together and then doubling that number. In this case $(7x)(4) = 28x$
 $28x$ doubled is $56x$.
 $56x$ is the same as the middle term in our original polynomial, therefore you are correct.

Questions to try:

- $81x^2 - 54x + 9$ $(9x - 3)^2$
- $144x^2 - 192x + 64$ $(12x - 8)^2$
- $49y^2 - 28y + 4$ $(7y - 2)^2$
- $25y^2 + 30y + 9$ $(5y + 3)^2$

Type 5: Binomial x Binomial

$$(a + b)(a - b)$$

Ex: $(x + 7)(x - 7) = x^2 - 49$

$(2y + 3)(2y - 3) = 4y^2 - 9$

Questions to try:

- $(x + 6)(x - 6) \quad x^2 - 36$
- $(3y + 5)(3y - 5) \quad 9y^2 - 25$
- $(2 + x)(2 - x) \quad 4 - x^2$
- $(2x + 5y)(2x - 5y) \quad 4x^2 - 25y^2$
- $(x + 2y)(x - 2y) \quad x^2 - 4y^2$

Type 5: Difference of Squares

Ex: $x^2 - 4 = (x + 2)(x - 2)$

$25y^2 - 16x^2 = (5y + 4x)(5y - 4x)$

Questions to try:

- $x^2 - 25 \quad (x - 5)(x + 5)$
- $100x^2 - 9 \quad (10x + 3)(10x - 3)$
- $49 - y^2 \quad (7 - y)(7 + y)$
- $16x^2 - 81y^2 \quad (4x - 9y)(4x + 9y)$
- $y^2 - 9x^2 \quad (y + 3x)(y - 3x)$

Type 6: Binomial x Binomial with 2 variables

Ex: $(5y + 2x)(y - 3x)$

$5y(y - 3x) + 2x(y - 3x)$

$5y^2 - 15xy + 2xy - 6x^2$

$5y^2 - 13xy - 6x^2$

Questions to try:

- $(3y + x)(2y - 3x) \quad 6y^2 - 7xy - 3x^2$
- $(x + 4y)(-x - y) \quad -x^2 - 5xy - 4y^2$
- $(7a - 7b)(3a + b) \quad 21a^2 - 14ab - 7b^2$

Type 6: Factoring Trinomials with Two Variables

Ex: $2a^2 - 7ab + 3b^2$

$2a^2 - 1ab - 6ab + 3b^2$

GCF of $2a^2 - 1ab$ is: a

GCF of $-6ab + 3b^2$ is: $-3b$

$a(2a - b) - 3b(2a - b)$

$(a - 3b)(2a - b)$

Use DECOMPOSITION!!

Two numbers that will multiply to give 6 and add to give -7 : $-1, -6$ Questions to try:

- $5x^2 - 13xy + 6y^2 \quad (5x - 3y)(x - 2y)$
- $3p^2 - 5pq - 2q^2 \quad (3p + q)(p - 2q)$
- $10x^2 - xy - 2y^2 \quad (5x + 2y)(2x - y)$

Combinations of Type 1 and Types 2, 3, 4, 5, 6

Find a GCF FIRST, then factor using one of the other methods.

Ex: $20x^2 + 70x + 60$

GCF of 10

$10(2x^2 + 7x + 6)$

$10(2x^2 + 4x + 3x + 6)$

$10[2x(x + 2) + 3(x + 2)]$

$10(2x + 3)(x + 2)$

Use DECOMPOSITION!!

Two numbers that will multiply to give 12 and add to give 7: $+4, +3$ Questions to try:

- $24y^2 - 72y + 54 \quad 6(2y - 3)(2y - 3) \quad \text{or} \quad 6(2y - 3)^2$
- $7x^2 - 35x + 42 \quad 7(x - 3)(x - 2)$
- $4y^2 - 20y - 56 \quad 4(y - 7)(y + 2)$
- $8y^2 - 72x^2 \quad 8(y - 3x)(y + 3x)$