Calculus 120 Exam Review Solutions 2017

(1) $y = \frac{7}{x+1}$ @ $x=3$, $y = \frac{7}{4}$

$m = f'(3)$

$f'(x) = 7(-[x+1]^{-2}] = -\frac{7}{(x+1)^2}$ @ $x=3$

$f'(3) = \frac{-7}{4^2} = -\frac{7}{16}$

$y - y_1 = m(x - x_1)$

$y - \frac{7}{4} = -\frac{7}{16}(x-3)$

$y = -\frac{7}{16}x + \frac{21}{16} + \frac{7}{4} = -\frac{7}{16}x + \frac{21}{16} + \frac{28}{16}$

$\boxed{y = -\frac{7}{16}x + \frac{49}{16}}$

(2) $f(x) = x^3 + 4$, $a = 8$

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f'(8) = \lim_{h \to 0} \frac{f(8+h) - f(8)}{h} = \lim_{h \to 0} \frac{(8+h)^3 + 4 - 516}{h}$

$= \lim_{h \to 0} \frac{\cancel{8^3} + 3 \cdot 8^2 h + 3 \cdot 8 \cdot h^2 + h^3 + \cancel{4} - \cancel{516}}{h} = \lim_{h \to 0} \frac{192h + 24h^2 + h^3}{h}$

$= \lim_{h \to 0} 192 + 24h + h^2 = \boxed{192}$

(3) $y = 6\sin(2x+12)$

$y' = 6\cos(2x+12) \cdot 2 = 12\cos(2x+12)$

$y'' = 12(-\sin(2x+12)) \cdot 2 = \boxed{-24\sin(2x+12)}$

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(4) $[f(g(x))]' = f'(g(x)) \cdot g'(x)$ @ $x=4$

$[f(g(4))]' = f'(g(4)) \cdot g'(4)$

$= f'(3) \cdot (-5)$

$= 6(-5)$

$= \boxed{-30}$

(5) $\left(\sqrt{f(x)+g(x)}\right)' = \frac{1}{2}\left(f(x)+g(x)\right)^{-1/2} \cdot \left[f'(x)+g'(x)\right]$ @ $x=3$

$\left(\sqrt{f(3)+g(3)}\right)' = \frac{1}{2}\left(f(3)+g(3)\right)^{-1/2} \cdot \left[f'(3)+g'(3)\right]$

$= \frac{1}{2}(1+16)^{-1/2} \cdot \left[8+3\right] = \frac{1}{2\sqrt{17}}(11)$

$= \boxed{\frac{11}{2\sqrt{17}}}$

(6) $s = \sqrt{5+4t}$

$v = \frac{1}{2}(5+4t)^{-1/2} \cdot 4$

$= \frac{2}{\sqrt{5+4t}}$ @ $t=1$

$v = \frac{2}{\sqrt{9}} = \boxed{\frac{2}{3} \text{ m/s}}$

(7) $x^2 + y^2 = 8$

$2x + 2y \cdot y' = 0$

$y \cdot y' = -2x$

$\boxed{y' = -\frac{x}{y}}$

$y'' = \frac{(-1)(y) - (-x)(y')}{y^2}$

$= \frac{-y + x(-x/y)}{y^2} \cdot \frac{y}{y}$

$= \frac{-y^2 - x^2}{y^3} = \frac{-(y^2+x^2)}{y^3}$

$\boxed{y'' = -\frac{8}{y^3}}$

by Staples

(8) $\lim_{x\to 0} \frac{\cos 9x - 1}{x^2} = \frac{1-1}{0} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\cos 9x - 1}{x^2} = \lim_{x\to 0} \frac{(-\sin(9x)\cdot 9)}{2x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\cos 9x - 1}{x^2} = \lim_{x\to 0} \frac{-9\sin(9x)}{2x} = \lim_{x\to 0} \frac{-9(\cos(9x)\cdot 9)}{2} = \boxed{\frac{-81}{2}}$

(9) $\lim_{t\to 0} \frac{\cos 7t - 1}{e^t - t - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

$\lim_{t\to 0} \frac{\cos 7t - 1}{e^t - t - 1} = \lim_{t\to 0} \frac{(-\sin(7t)\cdot 7)}{e^t - 1} = \frac{0}{1-1} = \frac{0}{0}$

$\lim_{t\to 0} \frac{\cos 7t - 1}{e^t - t - 1} = \lim_{t\to 0} \frac{-7\sin(7t)}{e^t - 1} = \lim_{t\to 0} \frac{-7\cos(7t)\cdot 7}{e^t} = \frac{-49}{1} = \boxed{-49}$

(10) $\lim_{\theta\to \frac{\pi}{6}} \frac{1+\cos 6\theta}{1-\sin 3\theta} = \frac{1+(-1)}{1-1} = \frac{0}{0}$

$\lim_{\theta\to \frac{\pi}{6}} \frac{1+\cos 6\theta}{1-\sin 3\theta} = \lim_{\theta\to \frac{\pi}{6}} \frac{-(\sin 6\theta)6}{-(\cos 3\theta)\cdot 3} = \frac{0}{0}$

$\lim_{\theta\to \frac{\pi}{6}} \frac{1+\cos 6\theta}{1-\sin 3\theta} = \lim_{\theta\to \frac{\pi}{6}} \frac{-6\sin 6\theta}{-3\cos 3\theta} = \lim_{\theta\to \frac{\pi}{6}} \frac{-6(\cos 6\theta)\cdot 6}{-3(-\sin 3\theta)\cdot 3} = \frac{-36(-1)}{9} = \boxed{4}$

(11) $\lim_{x\to 0^+} \frac{\ln(x^3+10x)}{\ln 3x} = \frac{\ln(0^+)}{\ln(0^+)} = \frac{\infty}{\infty}$

$\lim_{x\to 0^+} \frac{\ln(x^3+10x)}{\ln(3x)} = \lim_{x\to 0^+} \frac{\left(\frac{1}{x^3+10x}\right)\cdot(3x^2+10)}{\left(\frac{1}{3x}\right)\cdot 3} = \lim_{x\to 0^+} \frac{3x^2+10}{x^2+10} = \frac{10}{10} = \boxed{1}$

(12) $\lim_{t\to 1} \frac{t-1}{\ln t - \tan \pi t} = \frac{1-1}{0-0} = \frac{0}{0}$

$\lim_{t\to 1} \frac{t-1}{\ln t - \tan \pi t} = \lim_{t\to 1} \frac{1}{\frac{1}{t} - \sec^2(\pi t)\cdot \pi} = \boxed{\frac{1}{1-\pi}}$

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(13) $y = (\cos x)^x$

$\ln y = \ln(\cos x^x)$

$\ln y = x \cdot \ln(\cos x)$

$\frac{1}{y} \cdot y' = (1) \ln(\cos x) + x \cdot \frac{1}{\cos x}(-\sin x)$

$\frac{1}{y} \cdot y' = \ln \cos x - x \tan x$

$y' = y[\ln \cos x - x \tan x]$

$\boxed{y' = (\cos x)^x [\ln \cos x - x \tan x]}$

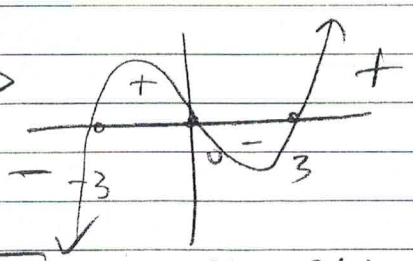
(14) $y = x^4 - 18x^2 + 9$

$y' = 4x^3 - 36x$

$0 = 4x(x^2 - 9)$

$x = 0, \pm 3$

$y' \Rightarrow$



$\boxed{\text{absolute (local) min @ } x = \pm 3 \quad (\pm 3, -72)}$

$y = 81 - 18(9) + 9$
$= -72$

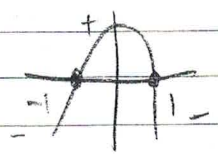
$\boxed{\text{local max @ } x = 0 \quad (0, 9)}$ $y = 9$

(15) $h(x) = \frac{3x}{x^2+1}$

$h'(x) = \frac{3(x^2+1) - 3x(2x)}{(x^2+1)^2} = \frac{3x^2 + 3 - 6x^2}{(x^2+1)^2} = \frac{3 - 3x^2}{(x^2+1)^2}$

$= \frac{3(1-x)(1+x)}{(x^2+1)^2}$

$1 - x^2 \Rightarrow$



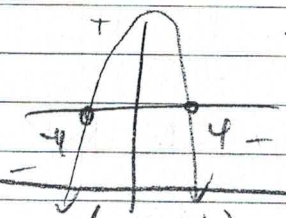
absolute (local)	min @ $x = -1$	$y = \frac{-3}{2}$
absolute (local)	max @ $x = 1$	$y = \frac{3}{2}$

(16) $f(x) = 48x - x^3$

$f'(x) = 48 - 3x^2$

$= 3(16 - x^2)$

$f'(x) = 3(4-x)(4+x)$



$f'(x) > 0$ on $x \in (-4, 4)$

$f'(x) < 0$ on $x \in (-\infty, -4) \cup (4, \infty)$

$$\boxed{\begin{array}{l} f(x) \text{ is increasing on } x \in (-4, 4) \\ f(x) \text{ is decreasing on } x \in (-\infty, -4) \cup (4, \infty) \end{array}}$$

(17) $f(x) = x\sqrt{9-x}$

$f'(x) = (1)\sqrt{9-x} + x \cdot \frac{1}{2}(9-x)^{-1/2}(-1) = \sqrt{9-x} - \frac{x}{2\sqrt{9-x}}$

$= \frac{2(9-x) - x}{2\sqrt{9-x}} = \frac{18 - 2x - x}{2\sqrt{9-x}} = \frac{18 - 3x}{2\sqrt{9-x}}$

$\sqrt{9-x} > 0$ for $9 - x > 0$

$-x > -9$

$x < 9$

$f'(x) > 0$ when $18 - 3x > 0$

$18 > 3x$

$6 > x$ or $x < 6$

$f'(x) < 0$ when $18 - 3x < 0$

$18 < 3x$

$6 < x$ or $6 < x < 9$

$$\boxed{\begin{array}{l} f(x) \text{ is increasing on } x \in (-\infty, 6) \\ f(x) \text{ is decreasing on } x \in (6, 9) \end{array}}$$

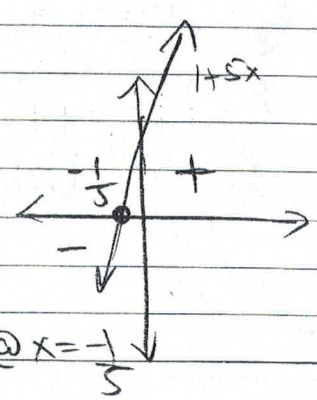
(18) $y = xe^{5x}$

$y' = (1)e^{5x} + xe^{5x} \cdot 5$

$y' = e^{5x}(1 + 5x)$

$e^{5x} > 0$

$\therefore$ sign changes for y' depend on $1 + 5x$



y' changes from $-$ to $+$ @ $x = -\frac{1}{5}$, so by the FDT there is a local min @ $x = -\frac{1}{5}$

(18) cont'd $x = \frac{-1}{5}$ $y = \frac{-1}{5} e^{(-1/5)\cdot -5} = \frac{-1}{5e}$

look @ end behavior: $\lim_{x \to \infty} xe^{5x} = \infty \cdot e^{\infty} = \infty$

$\lim_{x \to -\infty} xe^{5x} = \frac{(-\infty)}{e^{\infty}} = 0$

* since the graph tends towards $y=0$ on the left, $\left(\frac{-1}{5}, \frac{-1}{5e}\right)$ is an absolute minimum

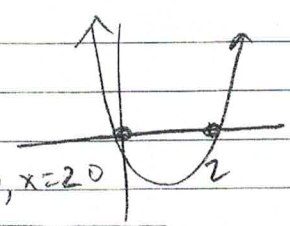
since the graph increases indefinitely on the right, there is no absolute maximum

(19) $y = \frac{1}{2}x^4 - 2x^3 + 12$

$y' = \frac{4}{2}x^3 - 6x^2 = 2x^3 - 6x^2$

$y'' = 6x^2 - 12x$

$= 6x(x-2)$



y'' changes sign @ $x=0, x=2$

POI occur @ $x=0$, $x=2$
$(0,12)$ $(2,4)$

(20) $y = xe^{17x}$

$y' = 1 \cdot e^{17x} + xe^{17x} \cdot 17$

$y' = e^{17x}(1+17x)$

$y'' = e^{17x} \cdot 17(1+17x) + e^{17x}(17)$

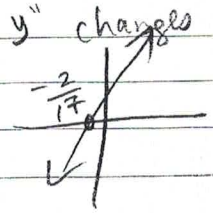
$= 17e^{17x}(1+17x+1)$

$y'' = 17e^{17x}(17x+2)$

$17 > 0$

$e^{17x} > 0$

y'' changes sign based on $17x+2$



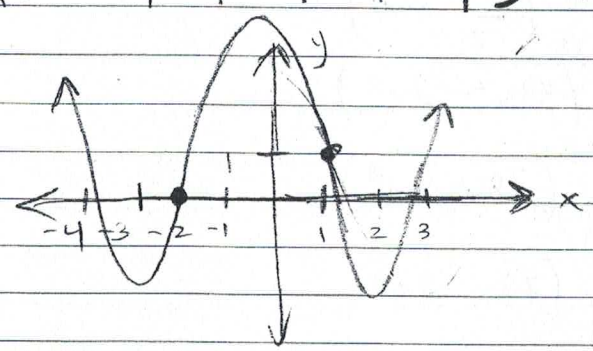
There is a poi @ $x = \frac{-2}{17}$, $y = \frac{-2}{17}e^{-2}$

$\left(\frac{-2}{17}, \frac{-2}{17e^2}\right)$

(21) $x = -3, -1, 2, -2, 1$

intervals	$f'(x)$	$f''(x)$	$f(x)$
$x < -3$	$-$	$+$	
$-3 < x < -2$	$+$	$+$	
$-2 < x < -1$	$+$	$-$	
$-1 < x < 1$	$-$	$-$	
$1 < x < 2$	$-$	$+$	
$x > 2$	$+$	$+$	

min @ $f(-3)$
poi @ $f(-2) \rightarrow (-2, 0)$
max @ $f(-1)$
poi @ $f(1) \rightarrow (1, 1)$
min @ $f(2)$

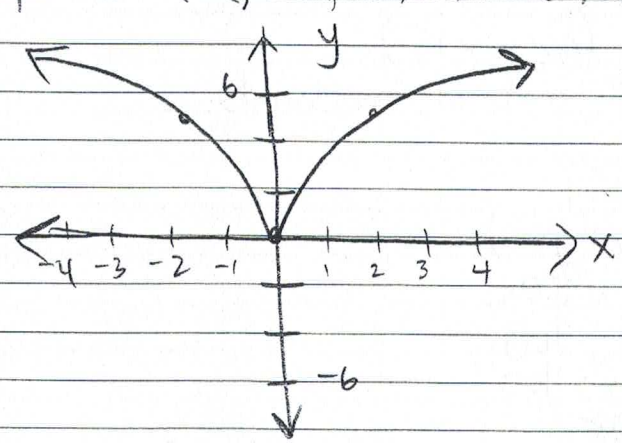


(22) (*) Cusp or vertical tangent @ $x = 0$

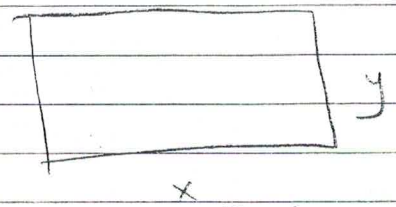
intervals	$f'(x)$	$f''(x)$	$f(x)$
$x < 0$	$-$	$-$	
$x > 0$	$+$	$-$	

min @ $x = 0 \rightarrow$ cusp

points: $(-2, 5)$, $(2, 5)$ & $(0, 0)$



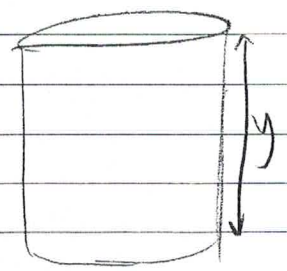
(23)



$P = 39cm$

$x = 2\pi r$

$r = \frac{x}{2\pi}$



$39 = 2x + 2y$

$2y = 39 - 2x$

$y = \frac{39}{2} - x = h$

$V = \pi r^2 h$

$V = \pi \left(\frac{x}{2\pi}\right)^2 \cdot \left(\frac{39}{2} - x\right)$

$V = \frac{x^2}{4\pi}\left(\frac{39}{2} - x\right)$

$= \frac{1}{8\pi}(39x^2 - 2x^3)$

$V' = \frac{1}{8\pi}(78x - 6x^2)$

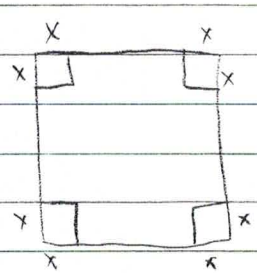
$0 = \frac{1}{8\pi}(78x - 6x^2)$

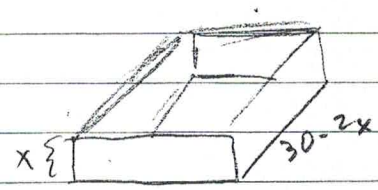
$0 = x(26 - 2x)$

$x = 0,\ x = 13\ cm$ $\quad y = \frac{39}{2} - 13 = \frac{39-26}{2} = \frac{13}{2}$

$\boxed{x = 13cm \text{ and } y = \frac{13}{2}\ cm \text{ give the largest volume.}}$

(24)





$(0 < x < 15)$ natural constraints

$30 - 2x$

$V = x(30-2x)^2$

$= x(900 - 120x + 4x^2)$

$V = 4x^3 - 120x^2 + 900x$

$V' = 12x^2 - 240x + 900$

$0 = 12x^2 - 240x + 900$

$0 = x^2 - 20x + 75$

$0 = (x-15)(x-5)$ $\quad V = 5(30-10)^2 = 2000 in^3$

$x = 15,\ x = 5$

$\boxed{20in \times 20in \times 5in \text{ gives the maximum volume, the max volume is } 2000 in^3.}$

(25) $f(x) = \sqrt{8x+9}$ $\quad a = 0$

$f(0) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2}(8x+9)^{-1/2} \cdot 8 = \frac{4}{\sqrt{8x+9}}$

$f'(0) = \frac{4}{3}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 3 + \frac{4}{3}(x-0)$

$\boxed{L(x) = \frac{4}{3}x + 3}$

(26) $(1.0004)^{50} \approx 1 + 50\left(\frac{4}{10000}\right) = 1 + \frac{1}{50} = \frac{51}{50}$

(27) $\sqrt[3]{1.003} \approx 1 + \frac{1}{3}(0.003) = 1 + 0.001 = 1.001$

(28) $(1-x)^4 \approx 1 - 4x$

(29) $\sqrt{99} \rightarrow$ use $\sqrt{x}$ near $a = 100$

$f(100) = 10$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(100) = \frac{1}{20}$

$L(x) = 10 + \frac{1}{20}(x - 100)$

$= 10 + \frac{x}{20} - 5 = \frac{x}{20} + 5$

$L(99) = \frac{99}{20} + 5 = \frac{199}{20}$

$\boxed{\therefore \sqrt{99} \cong \frac{199}{20}}$

(30) $y = x\sqrt{7x+2}$

$dy = f'(x)dx$

$dy = \left[(1)\sqrt{7x+2} + x \cdot \frac{1}{2\sqrt{7x+2}} \cdot 7\right]dx$

$\boxed{dy = \left[\sqrt{7x+2} + \frac{7x}{2\sqrt{7x+2}}\right]dx}$

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(31) $y = e^{\sin 3x}$

$dy = f'(x)dx$

$dy = e^{\sin 3x}(\cos 3x)\cdot 3\,dx$ @ $x = 2\pi$ $dx = 0.1$

$dy = e^{\sin 6\pi}\cdot \cos(6\pi)\cdot 3\cdot(0.1)$

$= e^{0}\cdot(1)\cdot 3\cdot(0.1)$

$\boxed{dy = 0.3}$

(32) $y = 6\sec\left(1-\frac{x}{6}\right)$

$dy = f'(x)dx$

$dy = \cancel{6}\sec\left(1-\frac{x}{6}\right)\tan\left(1-\frac{x}{6}\right)\cdot\left(\frac{-1}{\cancel{6}}\right)dx$ @ $x = 2, dx = 0.1$

$dy = -\sec\left(\frac{2}{3}\right)\tan\left(\frac{2}{3}\right)(0.1)$

$\boxed{dy = -0.127399442}$

(33) $5y = x^2 - 3xy$

$5y + 3xy = x^2$

$y(5+3x) = x^2$

$y = \frac{x^2}{5+3x}$

$dy = f'(x)dx$

$dy = \left(\frac{2x(5+3x) - x^2(3)}{(5+3x)^2}\right)dx$

$= \left(\frac{10x + 6x^2 - 3x^2}{(5+3x)^2}\right)dx$

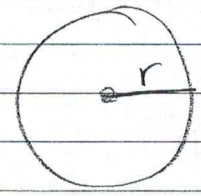
$dy = \frac{3x^2 + 10x}{(5+3x)^2}dx$ $x = 2$ $dx = -0.1$

$dy = \frac{3(2)^2 + 10(2)}{(5+3(2))^2}(-0.1)$

$= \frac{12+20}{(11)^2}(-0.1) = \frac{32}{121}(-0.1) = -\frac{3.2}{121}$

$\boxed{dy = \frac{-16}{605}}$ $= -0.02644\ldots$

(34) $\frac{dA}{dt} = 5\,mm^2/s$, $\frac{dr}{dt} = ?$ when $r = 145\,mm$

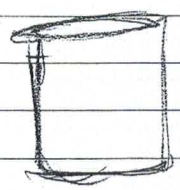


$A = \pi r^2$

$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$

$\frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{dr}{dt} \longrightarrow \frac{dr}{dt} = (5) \cdot \frac{1}{2\pi(145)} = \boxed{\frac{1}{58\pi}\ mm/s}$

(35) $\frac{dr}{dt} = 7\ in/s$, $\frac{dh}{dt} = -3\,in/s$, $\frac{dV}{dt} = ?$ when $r = 17$, $h = 5$



$V = \pi r^2 h$

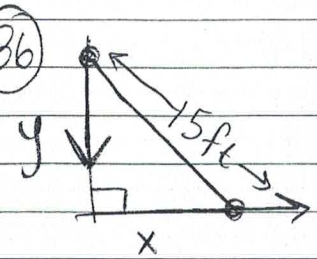
$\frac{dV}{dt} = \pi\left(2r \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt}\right)$

$= \pi\left(2(17)(7)(5) + (17)^2(-3)\right)$

$= \pi(1190 - 867)$

$\boxed{\frac{dV}{dt} = -323\pi\ in^3/s}$

(36)



$\frac{dy}{dt} = -2\,ft/s$

$\frac{dx}{dt} = ?$ when $x = 12\,ft$

$y = \sqrt{15^2 - 12^2}$
$= 9$

$x^2 + y^2 = 15^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$\frac{dx}{dt} = -\frac{y}{x} \cdot \frac{dy}{dt} = -\frac{9}{12}(-2)$

$\boxed{\frac{dx}{dt} = \frac{3}{2}\ ft/s}$

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(37) $y = \int_1^{\sqrt{x}} 16t^9\,dt$ $\qquad \frac{d}{dx}\int_a^x f(t)\,dx = f(x)$

$$\frac{dy}{dx} = \frac{d}{dx}\int_1^{\sqrt{x}} 16t^9\,dt = 16(\sqrt{x})^9 \cdot \frac{1}{2\sqrt{x}} = 8(\sqrt{x})^8 = \boxed{8x^4}$$

(38) $\frac{dy}{dx} = \csc^2 x - 36x^5$

$y = -\cot x - \frac{36x^6}{6} + C$

$\boxed{y = -\cot x - 6x^6 + C}$

(39) $\frac{dy}{dt} = \cos t - e^{-t} - 35t^6$

$y = \sin t - \frac{e^{-t}}{(-1)} - \frac{35t^7}{7} + C$

$\boxed{y = \sin t + e^{-t} - 5t^7 + C}$

(40) $\frac{dy}{du} = u^5 - \frac{1}{u^5} = u^5 - u^{-5}$

$y = \frac{u^6}{6} - \frac{u^{-4}}{(-4)} + C$

$\boxed{y = \frac{1}{6}u^6 + \frac{1}{4u^4} + C}$

(41) $\frac{dy}{dx} = \sin(4x + \pi)$ $\qquad y=5,\ x=0$

$y = -\frac{\cos(4x+\pi)}{4} + C$

$5 = -\frac{\cos(\pi)}{4} + C$

(41) cont'd $5 = \frac{1}{4} + C$

$5 - \frac{1}{4} = C$

$\frac{19}{4} = C$

$$\boxed{y = \frac{-\cos(4x+\pi)}{4} + \frac{19}{4}}$$

(42) $\frac{dv}{dt} = 4\sec t \tan t + 9t - 9e^t$ $\quad v = 7,\ t = 0$

$v = 4\sec t + \frac{9}{2}t^2 - 9e^t + C$

$7 = 4\sec(0) + \frac{9}{2}(0)^2 - 9e^0 + C$

$7 = 4 - 9 + C$

$7 + 5 = C$

$C = 12$

$$\boxed{V = 4\sec t + \frac{9}{2}t^2 - 9e^t + 12} \text{ on } t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

(43) $\frac{dy}{dx} = 6x^2 - 4x + 18$ $\quad y = 25,\ x = 1$

$y = \frac{6}{3}x^3 - \frac{4x^2}{2} + 18x + C$

$y = 2x^3 - 2x^2 + 18x + C$

$25 = 2 - 2 + 18 + C$

$7 = C$

$$\boxed{y = 2x^3 - 2x^2 + 18x + 7}$$

(44) $y' = \frac{y}{9+x}$ $\quad y = 18,\ x = 0$

$\frac{1}{y}\,dy = \frac{dx}{9+x}$

$\int \frac{1}{y}\,dy = \int \frac{dx}{9+x}$

$\ln|y| = \ln|x+9| + C$

$e^{\ln|y|} = e^{\ln|x+9| + C}$

$y = (x+9)\cdot e^C$

$18 = 9 \cdot e^C$

$2 = e^C$

$$\boxed{y = 2(x+9)}$$

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(45) $\frac{dy}{dx} = \frac{9x^2}{\sqrt{y}}$ $\qquad y=1,\ x=0$

$\sqrt{y}\,dy = 9x^2 dx$

$\int \sqrt{y}\,dy = \int 9x^2 dx$

$\frac{2}{3}y^{3/2} = \frac{9}{3}x^3 + C$

$\frac{2}{3} = 0 + C$

$\frac{2}{3} = C$

$\left(\frac{2}{3}y^{3/2} = 3x^3 + \frac{2}{3}\right)\cdot\frac{3}{2}$

$y^{3/2} = \frac{9}{2}x^3 + 1$

$\boxed{y = \left(\frac{9}{2}x^3 + 1\right)^{2/3}}$

(46) $\int_{-1}^{2}(x^2+2x-4)\,dx$ $\qquad a=-1,\ b=2$ $\qquad x_i = -1 + \frac{3i}{n}$

$\Delta x = \frac{b-a}{n} = \frac{3}{n}$

$$\int_{-1}^{2}(x^2+2x-4)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(-1+\frac{3i}{n}\right)^2 + 2\left(-1+\frac{3i}{n}\right) - 4\right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\left[\frac{3}{n}\sum_{i=1}^{n}\left(1 - \cancel{\frac{6i}{n}} + \frac{9i^2}{n^2} - 2 + \cancel{\frac{6i}{n}} - 4\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{n}\left(\frac{9}{n^2}\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 5\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{n}\left(\frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} - 5n\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{9}{2}\left(\frac{2n^3+3n^2+n}{n^3}\right) - 15\right]$$

$$= \frac{9}{2}\cdot 2 - 15 = \boxed{-6}$$

(47) $\int_{3}^{5}(x^2-4x)\,dx$ $\qquad a=3,\ b=5$

$\Delta x = \frac{2}{n}$ $\qquad x_i = 3 + \frac{2i}{n}$

(47) cont'd

$$\int_3^5 (x^2-4x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(3+\frac{2i}{n}\right)^2 - 4\left(3+\frac{2i}{n}\right)\right]\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\left(\sum_{i=1}^{n}\left(9+\frac{12i}{n}+\frac{4i^2}{n^2}-12-\frac{8i}{n}\right)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\left(\frac{4}{n^2}\sum_{i=1}^{n} i^2 + \frac{4}{n}\sum_{i=1}^{n} i - \sum_{i=1}^{n} 3\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\left(\frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{4}{n}\cdot\frac{n(n+1)}{2} - 3n\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{3}\left(\frac{2n^3+3n^2+n}{n^3}\right) + 4\left(\frac{n^2+n}{n^2}\right) - 6\right]$$

$$= \frac{4}{3}\cdot 2 + 4 - 6 = \frac{8}{3} - 2 = \boxed{\frac{2}{3}}$$

(48) $\int_0^4 (x^3-x)\,dx$ $\quad a=0,\ b=4 \quad \Delta x = \frac{4}{n} \quad x_i = 0+\frac{4i}{n} = \frac{4i}{n}$

$$\int_0^4 (x^3-x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{4i}{n}\right)^3 - \left(\frac{4i}{n}\right)\right]\cdot\frac{4}{n}$$

$$= \lim_{n\to\infty}\left[\frac{4}{n}\left(\sum_{i=1}^{n}\left(\frac{64i^3}{n^3} - \frac{4i}{n}\right)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n}\left(\frac{64}{n^3}\sum_{i=1}^{n} i^3 - \frac{4}{n}\sum_{i=1}^{n} i\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n}\left(\frac{64}{n^3}\left[\frac{n(n+1)}{2}\right]^2 - \frac{4}{n}\left(\frac{n(n+1)}{2}\right)\right)\right]$$

$$= \lim_{n\to\infty}\left[64\left(\frac{n^4+2n^3+n^2}{n^4}\right) - 8\left(\frac{n^2+n}{n^2}\right)\right]$$

$$= 64 - 8 = \boxed{56}$$

(49) $\lim_{n\to\infty}\sum_{k=1}^{n} c_k^{\,7}\,\Delta x_k,\ [-1,5] = \boxed{\int_{-1}^{5} x^7\,dx}$

(50) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3c_k^2 - 11c_k + 19\right)\Delta x_k,\ [-5,4] = \boxed{\int_{-5}^{4}(3x^2-11x+19)\,dx}$

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(51) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{11-9c_k^2} \Delta x_k, [3,5] = \boxed{\int_3^5 \frac{4}{11-9x^2}\,dx}$

(52) $\int_3^5 f(x)dx = -4$

$\int_5^5 f(x)dx = \boxed{0}$ $\qquad \int_5^3 f(x)dx = -\int_3^5 f(x)dx = -(-4) = \boxed{4}$

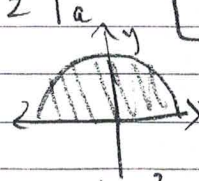
(53) $\int_6^{10} f(x)dx = -6 \qquad \int_6^{10} g(x)dx = 9$

$$\int_6^{10} [4f(x)+g(x)]dx = 4\int_6^{10} f(x)dx + \int_6^{10} g(x)dx$$
$$= 4(-6) + 9$$
$$= \boxed{-15}$$

(54) $\int_{-4}^{4} f(z)dz = 0 \qquad \int_{-4}^{7} f(z)dz = 3$

$$\int_7^4 f(x)dx = -\int_4^7 f(x)dx = -\left[\int_4^{-4} f(x)dx + \int_{-4}^{7} f(x)dx\right]$$
$$= \int_{-4}^{4} f(x)dx - \int_{-4}^{7} f(x)dx$$
$$= 0 - 3$$
$$= \boxed{-3}$$

(55) $\int_a^b 7x\,dx = \frac{7x^2}{2}\Big|_a^b = \boxed{\frac{7(b^2-a^2)}{2}}$

(56) $\int_{-4}^{4} \sqrt{16-x^2}\,dx =$ 

area under a semicircle (r=4)

$= \frac{1}{2}\pi(4)^2 = \boxed{8\pi}$

(57) $\int_{-2}^{-1} 3x^{-4}\,dx = \frac{3x^{-3}}{(-3)}\Big|_{-2}^{-1} = \frac{-1}{x^3}\Big|_{-2}^{-1} = \left(\frac{-1}{(-1)^3}\right) - \left(\frac{-1}{(-2)^3}\right)$

$= 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

(58) $\int_{-\frac{\pi}{2}}^{\pi/2} (10+\cos x)\,dx = 10x + \sin x \Big|_{-\pi/2}^{\pi/2} = \left(10\cdot\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \left(10\left(\frac{-\pi}{2}\right) + \sin\left(\frac{-\pi}{2}\right)\right)$

$= 5\pi + 1 - (-5\pi) - (-1)$

$= \boxed{10\pi + 2}$

(59) $\int (3\sqrt{x}+6)\,dx = \frac{3x^{3/2}}{(3/2)} + 6x + C = \boxed{2x^{3/2} + 6x + C}$

(60) $\int (13x^{-7} + \csc^2 x)\,dx = \frac{13x^{-6}}{(-6)} - \cot x + C = \boxed{-\frac{13}{6x^6} - \cot x + C}$

(61) $\int \left(x^{-4} + \frac{1}{3\sqrt{x}}\right)dx = \int \left(x^{-4} + \frac{1}{3}x^{-1/2}\right)dx = \frac{x^{-3}}{(-3)} + \frac{1}{3}\cdot\frac{x^{1/2}}{(1/2)} + C$

$= \boxed{-\frac{1}{3x^3} + \frac{2\sqrt{x}}{3} + C}$

(62) $\int \frac{x}{5x^2-6}\,dx$

$u = 5x^2 - 6$

$du = 10x\,dx$

$\frac{1}{10}du = x\,dx$

$\int \frac{x}{5x^2-6}\,dx = \int \frac{1}{10}\cdot\frac{1}{u}\,du = \frac{1}{10}\int \frac{1}{u}\,du = \frac{1}{10}\ln|u| + C$

$= \boxed{\frac{1}{10}\ln|5x^2-6| + C}$

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(63) $\int \frac{dx}{x^2+25} = \int \frac{1}{25} \cdot \frac{1}{\left(\frac{x^2}{25}+1\right)} dx = \frac{1}{25} \int \frac{dx}{\left(\frac{x}{5}\right)^2+1}$

$u = \frac{x}{5}$

$du = \frac{1}{5} dx$

$5du = dx$

$= \frac{1}{25} \int \frac{5du}{u^2+1} = \frac{1}{5} \int \frac{du}{u^2+1}$

$= \frac{1}{5} \arctan u + C$

$= \boxed{\frac{1}{5} \arctan\left(\frac{x}{5}\right) + C}$

(64) $\int \frac{\sin t}{(3+\cos t)^3} dt$

$u = 3+\cos t$

$du = -\sin t dt$

$-du = \sin t dt$

$\int \frac{\sin t}{(3+\cos t)^3} dt = \int \frac{-du}{u^3} = -\int u^{-3} du = -\frac{u^{-2}}{(-2)} + C$

$= \boxed{\frac{1}{2(3+\cos t)^2} + C}$

(65) $\int \frac{6x-24}{x^2-4x-32} dx = 6 \int \frac{x-4}{(x-8)(x+4)} dx$

$\frac{x-4}{(x-8)(x+4)} = \frac{A}{x-8} + \frac{B}{x+4}$

$x-4 = A(x+4) + B(x-8)$

$x-4 = Ax + 4A + Bx - 8B$

$x-4 = (A+B)x + (4A-8B)$

$(A+B=1) \times 4$

$4A - 8B = -4$

$-(4A + 4B = 4)$

$-12B = -8$

$B = 8/12 = 2/3$

$A = \frac{1}{3}$

$\frac{x-4}{x^2-4x-32} = \frac{1}{3(x-8)} + \frac{2}{3(x+4)}$

(65) cont'd

$$6\int \frac{(x-4)}{x^2-4x-32}\,dx = 6\int\left(\frac{1}{3(x-8)} + \frac{2}{3(x+4)}\right)dx = \int\left(\frac{2}{x-8} + \frac{4}{x+4}\right)dx$$

$$= 2\ln|x-8| + 4\ln|x+4| + C$$

$$= \boxed{\ln\left(|x-8|^2 \cdot |x+4|^4\right) + C}$$

(66) $$\int \frac{x+9}{x^2+2x}\,dx = \int \frac{x+9}{x(x+2)}\,dx$$

$$\frac{x+9}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$x+9 = A(x+2) + Bx$$

$$x+9 = Ax + 2A + Bx$$

$$x+9 = (A+B)x + 2A$$

$$A+B=1 \qquad 2A=9$$

$$A = \frac{9}{2} \qquad B = 1 - \frac{9}{2} = \frac{-7}{2}$$

$$\int \frac{x+9}{x^2+2x}\,dx = \int\left(\frac{9}{2x} - \frac{7}{2(x+2)}\right)dx = \frac{9}{2}\ln|x| - \frac{7}{2}\ln|x+2| + C$$

$$= \frac{1}{2}\left(9\ln|x| - 7\ln|x+2|\right) + C$$

$$= \boxed{\frac{1}{2}\left(\ln\left|\frac{x^9}{(x+2)^7}\right|\right) + C}$$

(67) $$\int \tan^2 3x\,dx = \int\left(\sec^2(3x) - 1\right)dx$$

$$= \boxed{\frac{\tan(3x)}{3} - x + C}$$

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(68) $\int \frac{dx}{\sin^2 8x} = \int \csc^2(8x)dx = \boxed{\frac{-\cot(8x)}{8} + C}$

(69) $\int \cos^3(8x)dx = \int \cos^2(8x)\cdot\cos(8x)dx = \int (1-\sin^2(8x))\cdot\cos(8x)dx$

$$\left\{\begin{array}{l} u = \sin(8x) \\ du = 8\cos(8x)dx \\ \frac{1}{8}du = \cos(8x)dx \end{array}\right\}$$

$= \int (1-u^2)\frac{1}{8}du = \frac{1}{8}\int (1-u^2)du$

$= \frac{1}{8}\left(u - \frac{u^3}{3}\right) + C$

$= \boxed{\frac{1}{8}\left(\sin(8x) - \frac{\sin^3(8x)}{3}\right) + C}$

(70) $a(t) = 1.2 + 3\sqrt{t}$ mph/s

$V(0) = 0$

$V = \int_0^{16} (1.2 + 3\sqrt{t})dt = 1.2t + \frac{3t^{3/2}}{(3/2)}\Big|_0^{16} = 1.2t + 2t^{3/2}\Big|_0^{16}$

$= 147.2$ mph/s $\cdot$ s

$= \boxed{147.2 \text{ mph}}$

(71) $a(t) = 1.3 + 2\sqrt{t}$ mph/s

$V(0) = 0$

$V = \int_0^t (1.3 + 2\sqrt{x})dx = 1.3t + \frac{2t^{3/2}}{(3/2)} = 1.3t + \frac{4}{3}t^{3/2}$

$d = \int_0^4 \left(1.3t + \frac{4}{3}t^{3/2}\right)dt = \frac{1.3t^2}{2} + \frac{4}{3}\frac{t^{5/2}}{(5/2)}\Big|_0^4 = \frac{1.3t^2}{2} + \frac{8}{15}t^{5/2}\Big|_0^4$

$= 27.4\overline{6}\ \frac{\text{miles}}{\text{hr}} \cdot 8 \times \frac{1\text{ hr}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}}$

$= \boxed{0.0076 \text{ miles}}$

(72) $b'(t) = 50e^{0.3t}$ (town A)
$b'(t) = 89e^{0.4t}$ (town B)

$$\int_0^{10} 89e^{0.4t}\,dt - \int_0^{10} 50e^{0.3t}\,dt = \int_0^{10}\left(89e^{0.4t} - 50e^{0.3t}\right)dt$$

$$= \frac{89}{0.4}e^{0.4t} - \frac{50}{0.3}e^{0.3t}\Big|_0^{10}$$

$$= \frac{445}{2}e^{0.4t} - \frac{500}{3}e^{0.3t}\Big|_0^{10}$$

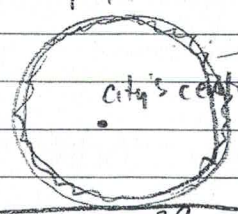
$$= 8800.499 - 55.833\ldots$$

$$= 8744.66\ldots$$

≈ 8745 more births in town B than town A

(73) $P = 8100(3.9 - r)$ ppl/square mile

population = population density × area



→ a thin slice has radius r and width Δr

* if we straighten this out, the area is $2\pi r \cdot \Delta r$; we want the area of infinitely many thin strips

$$\therefore \boxed{\text{population} = \int_0^{3.9} 8100(3.9-r)2\pi r\,dr}$$

(74) $C(t) = 4.3 - 2.1\sin\left(\frac{\pi t}{12}\right)$

average consumption = consumption rate × time

$$\text{average daily consumption} = \int_0^{24}\left(4.3 - 2.1\sin\left(\frac{\pi t}{12}\right)\right)dt$$

$$= 4.3t + 2.1\cos\left(\frac{\pi t}{12}\right)\cdot\frac{12}{\pi}\Big|_0^{24}$$

$$= (111.22\ldots) - (8.02\ldots) = \boxed{103.2 \text{ kWh}}$$

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(75) $F = kx$

$4 = k(0.05)$

$k = \frac{4}{0.05} = 80 \text{ N/m}$

$F = 80x$

$12 = 80x$

$x = \frac{12}{80} = \frac{3}{20}$

$W = \int_0^{3/20} 80x\,dx = 40x^2 \Big|_0^{3/20} = 40\left(\frac{3^2}{20^2}\right) - 0$

$= \frac{\cancel{2} \cdot 9}{\cancel{20}} = \boxed{\frac{9}{10} \text{ J}}$

(76) $W = \int_0^2 x\sqrt{4-x^2}\,dx$

$u = 4 - x^2$ $\quad x=0,\ u=4$

$du = -2x\,dx$ $\quad x=2,\ u=0$

$-\frac{1}{2}du = x\,dx$

$W = \int_4^0 -\frac{1}{2}\sqrt{u}\,du = \frac{1}{2}\int_0^4 u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{(3/2)}\Big|_0^4 = \frac{1}{3}u^{3/2}\Big|_0^4$

$= \frac{4^{3/2}}{3} = \boxed{\frac{8}{3} \text{ J}}$

(77) $A = \int_{-3}^0 \left(4x - (-x^3 + x^2 + 16x)\right)dx + \int_0^4 \left(-x^3 + x^2 + 16x - 4x\right)dx$

$= \int_{-3}^0 (x^3 - x^2 - 12x)\,dx + \int_0^4 (-x^3 + x^2 + 12x)\,dx$

$= \frac{x^4}{4} - \frac{x^3}{3} - 6x^2 \Big|_{-3}^0 + -\frac{x^4}{4} + \frac{x^3}{3} + 6x^2 \Big|_0^4$

$= (0) - \left(-\frac{99}{4}\right) + \left(\frac{160}{3}\right) - (0) = \frac{99}{4} + \frac{160}{3} = \frac{297}{12} + \frac{640}{12}$

$= \boxed{\frac{937}{12}}$

(78) $A = \int_0^1 2x^2\,dx + \int_1^{7/5} (7-5x)\,dx$

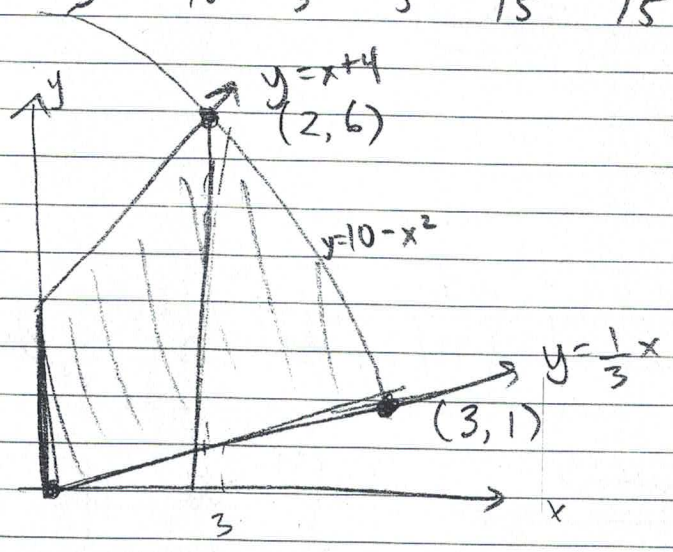
$7-5x=0$
$7=5x$
$\frac{7}{5}=x$

$= \frac{2x^3}{3}\Big|_0^1 + 7x - \frac{5x^2}{2}\Big|_1^{7/5}$

$= \left(\frac{2}{3}\right) - 0 + \left(\frac{49}{10}\right) - \frac{9}{2} \times \frac{5}{5}$

$= \frac{2}{3} + \frac{4}{10} = \frac{2}{3} + \frac{2}{5} = \frac{10}{15} + \frac{6}{15} = \boxed{\frac{16}{15}}$

(79)



$10-x^2 = x+4$
$0 = x^2+x-6$
$= (x+3)(x-2)$
$x=2$

$10-x^2 = \frac{1}{3}x$

$0 = x^2 + \frac{1}{3}x - 10$

$0 = 3x^2 + x - 30$
$0 = (3x+10)(3x-9)$
$0 = (3x+10)(x-3)$

$A = \int_0^2 \left(x+4-\frac{1}{3}x\right)dx + \int_2^3 \left(10-x^2-\frac{1}{3}x\right)dx$

$= \int_0^2 \left(\frac{2}{3}x+4\right)dx + \int_2^3 \left(-x^2-\frac{1}{3}x+10\right)dx$

$= \frac{1}{3}x^2 + 4x\Big|_0^2 + -\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^2}{2} + 10x\Big|_2^3$

$= \left(\frac{28}{3}\right) - 0 + \left(\frac{39}{2}\right) - \left(\frac{50}{3}\right) = \frac{39}{2} - \frac{22}{3} = \frac{117}{6} - \frac{44}{6} = \boxed{\frac{73}{6}}$

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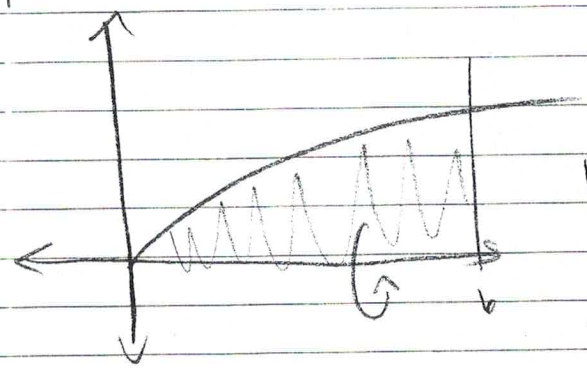
(80) $y=\sqrt{x}$, $y=0$, $x=0$, $x=6$

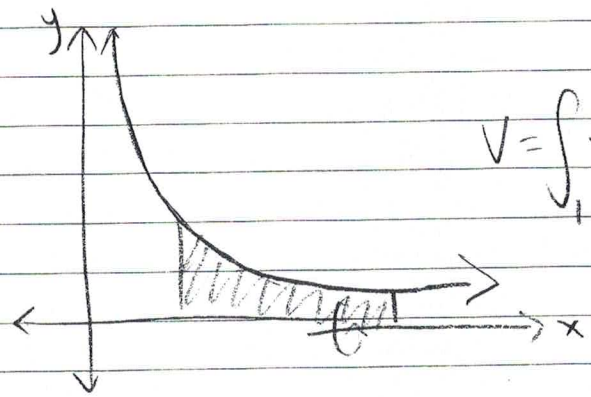


$$V=\int_0^6 \pi(\sqrt{x})^2\,dx = \pi\int_0^6 x\,dx$$

$$=\pi\cdot\frac{x^2}{2}\Big|_0^6 = \pi\left(\frac{36}{2}-0\right)$$

$$=\boxed{18\pi}$$

(81) $y=\frac{1}{x}$, $y=0$, $x=1$, $x=5$

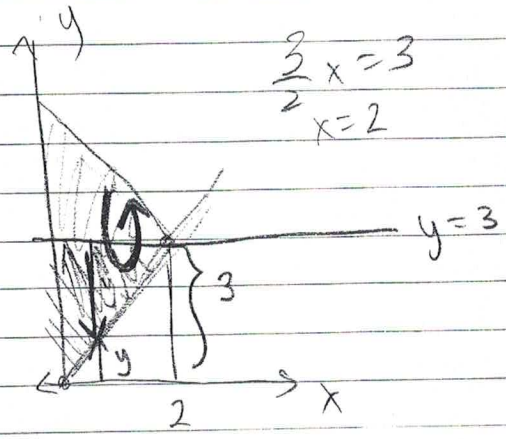


$$V=\int_1^5 \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^5 \frac{1}{x^2}\,dx$$

$$=\pi\int_1^5 x^{-2}\,dx = \pi\left[\frac{x^{-1}}{(-1)}\Big|_1^5\right]$$

$$=\pi\left[-\frac{1}{5}-\left(-\frac{1}{1}\right)\right]=\boxed{\frac{4\pi}{5}}$$

(82)



$\frac{3}{2}x=3$

$x=2$

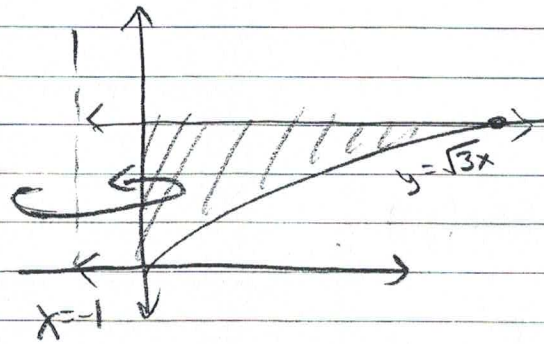
$$V=\int_0^2 \pi(3-y)^2\,dx = \int_0^2 \pi\left(3-\frac{3}{2}x\right)^2 dx$$

$$=\pi\int_0^2\left(9-9x+\frac{9}{4}x^2\right)dx$$

$$=\pi\left(9x-\frac{9}{2}x^2+\frac{3}{4}x^3\Big|_0^2\right)$$

$$=\pi\left[(6-0)\right]=\boxed{6\pi}$$

(83)



$y=3$ meets @ $y=3$

<u>washers:</u> $r_1 = 1$

$r_2 = 1 + x$, where x meets $y=\sqrt{3x}$

$= 1 + \frac{y^2}{3}$

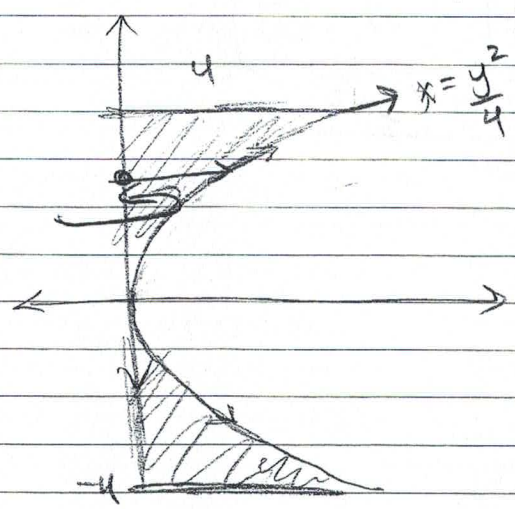
$$A = \pi\left[\left(1+\frac{y^2}{3}\right)^2 - 1^2\right] = \pi\left[\left(1+\frac{2}{3}y^2+\frac{y^4}{9}\right)-1\right]$$

$$= \pi\left[\frac{y^4}{9}+\frac{2}{3}y^2\right]$$

$$V = \int_0^3 \pi\left[\frac{y^4}{9}+\frac{2}{3}y^2\right]dy = \pi\left[\frac{y^5}{45}+\frac{2}{9}y^3\Big|_0^2\right]$$

$$= \pi\left[\frac{57}{5}-0\right] = \boxed{\frac{57\pi}{5}}$$

(84)



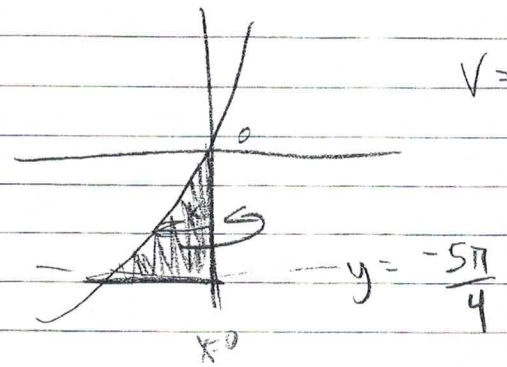
$r = x = \frac{y^2}{4}$

$$V = \int_{-4}^{4} \pi\left(\frac{y^2}{4}\right)^2 dy = \int_{-4}^{4} \pi\frac{y^4}{16}\,dy$$

$$= \frac{\pi}{16}\left[\frac{y^5}{5}\Big|_{-4}^{4}\right] = \frac{\pi}{16}\left(\frac{1024}{5}-\left(-\frac{1024}{5}\right)\right)$$

$$= \frac{\pi}{16}\cdot\frac{2048}{5} = \boxed{\frac{128\pi}{5}}$$

(85) $x = 2\tan\left(\frac{y}{5}\right)$, $x = 0$, $y = \frac{-5\pi}{4}$



$* \tan^2\theta = \sec^2\theta - 1$

$$V = \int_{-\frac{5\pi}{4}}^{0} \pi \left(2\tan\left(\frac{y}{5}\right)\right)^2 dy$$

$$= 4\pi \int_{-\frac{5\pi}{4}}^{0} \tan^2\left(\frac{y}{5}\right) dy$$

$$= 4\pi \int_{-\frac{5\pi}{4}}^{0} \left(\sec^2\left(\frac{y}{5}\right) - 1\right) dy$$

$$= 4\pi \left[\tan\left(\frac{y}{5}\right) \cdot 5 - y \Big|_{-\frac{5\pi}{4}}^{0} \right]$$

$$= 4\pi \left[(0 - 0) - \left(\tan\left(-\frac{\pi}{4}\right) \cdot 5 + \frac{5\pi}{4}\right) \right]$$

$$= 4\pi \left[-(-5) - \frac{5\pi}{4} \right]$$

$$= \boxed{20\pi - 5\pi^2}$$