Proving Identities

Example 1: Use the Conjugate to Prove an Identity

Prove $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$ for all permissible values of x.

$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$	
LHS	**RHS**
$\frac{1-\sin x}{\cos x}$	$\frac{\cos x}{1+\sin x}$

Example 2: Use a Common Denominator to Prove an Identity

Prove $\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2\tan^2 x$ for all permissible values of x.

$\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2\tan^2 x$	
LHS	**RHS**
$\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1}$	$2\tan^2 x$

Example 3: Use Factoring to Prove an Identity

Prove $\sec^4 x - 1 = \dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$ for all permissible values of x.

$\sec^4 x - 1 = \dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$	
LHS	**RHS**
$\sec^4 x - 1$	$\dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$

Example 4: Prove a More Complicated Identity

Prove $\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \dfrac{\cot x - 1}{\cot x}$ for all permissible values of x.

$\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \dfrac{\cot x - 1}{\cot x}$	
LHS	**RHS**
$\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$	$\dfrac{\cot x - 1}{\cot x}$