Transformations of Exponential Functions

- To graph an exponential function of the form $y = a(c)^{b(x-h)} + k$, apply transformations to the graph of the base function, $y = c^x$, where c > 0.

Example 1: Apply Transformations and Sketch a Graph

Consider the base function $y = 3^x$. For each transformed function,

- State the parameters and describe the corresponding transformations.
- Write the mapping rule.
- Graph the base function and the transformed function on the same grid.
- State the domain, range, intercepts, and equation of the horizontal asymptote.

a. $y = \frac{1}{3}(3)^{x+4}$ b. $y = 2(3)^{-2(x-1)} - 5$

Solution:

a. $y = \frac{1}{3}(3)^{x+4}$

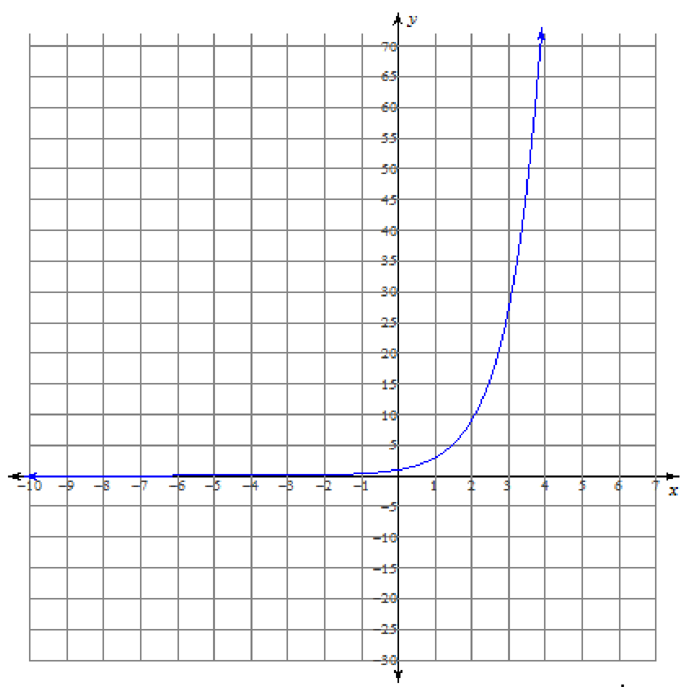
- Compare the function $y = \frac{1}{3}(3)^{x+4}$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

a = _____, which corresponds to a ______________________________ by a factor of _____.
b = _____, which corresponds to __________________________________.
h = _____, which corresponds to a ______________________________ of _____ units ______________.
k = _____, which corresponds to __________________________________.

- Mapping rule: _____________________________
- Complete each table of values and sketch the graph of the function $y = \frac{1}{3}(3)^{x+4}$.

$y = 3^x$	
x	y
–2	
–1	
0	
1	
2	
3	
4	

$y = \frac{1}{3}(3)^{x+4}$	
x	y



- For the function $y = \frac{1}{3}(3)^{x+4}$:

Domain: ______________ Range: ________________

x-intercept: __________ y-intercept: __________

Equation of the horizontal asymptote : __________

b. $y = 2(3)^{-2(x-1)} - 5$

- Compare the function $y = 2(3)^{-2(x-1)} - 5$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

  a = _____, which corresponds to a ______________________________ by a factor of _____.

  b = _____, which corresponds to a ______________________________ by factor of _____,

  and a _________________________ in the _______________.

  h = _____, which corresponds to a ______________________________ of _____ unit _______________.

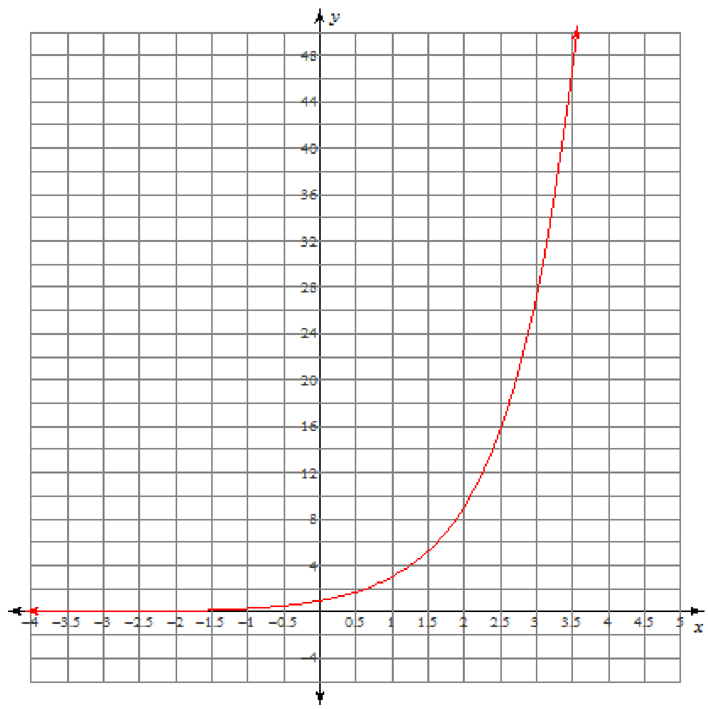
  k = _____, which corresponds to a ______________________________ of _____ units _______________.

- Mapping rule: ___________________________________

- Complete each table of values and sketch the graph of the function $y = 2(3)^{-2(x-1)} - 5$.

$y = 3^x$	
x	y
–2	
–1	
0	
1	
2	
3	
4	

$y = 2(3)^{-2(x-1)} - 5$	
x	y



- For the function $y = 2(3)^{-2(x-1)} - 5$:

  Domain: ______________ Range: _________________

  x-intercept: ___0.58___ y-intercept: __________

  Note – In the next unit, we will learn an algebraic method of solving exponential equations that will enable us to determine the value of the x-intercept.

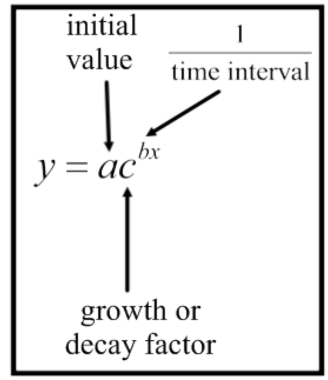
  Equation of the horizontal asymptote : __________

Example 2: Use Transformations of an Exponential Function to Model a Situation

An initial population of 2000 insects is expected to triple every 5 days.

a. Write an exponential function in the form $y = a(c)^{bx}$ to model this situation.
b. Use your equation to calculate the insect population in 21 days.

Solution:



a. Determine the exponential function $y = a(c)^{bx}$:

b. Insect population in 21 days:

Example 3: Use Transformations of an Exponential Function to Model a Situation

A hockey card that was purchased for $250 is expected to increase in value by 12% every 3 years.

a. Write an exponential function in the form $y = a(c)^{bx}$ to model this situation.
b. Use your equation to calculate the value of the card in 5 years.

Solution:

a. Determine the exponential function $y = a(c)^{bx}$:

b. Value of the card in 5 years: