Solving Exponential Equations

Exponential equation: *An equation that has a variable in an exponent*

Example 1: Rewriting Powers with a Specified Base

Rewrite each of the following powers with a base of 2.

a. 32 b. 16^3 c. $\sqrt[5]{8}$ d. $\left(\frac{1}{64}\right)^{\frac{1}{3}}$

Solution:

a. 32 = b. 16^3 = c. $\sqrt[5]{8}$ = d. $\left(\frac{1}{64}\right)^{\frac{1}{3}}$ =

Example 2: Solve an Exponential Equation by Using a Common Base

Solve each equation.

a. $10^{x+4} = 1000^{x-4}$ b. $8^{x+2} = \left(\frac{1}{4}\right)^{x+3}$ c. $\left(\sqrt{3}\right)^{3x} = 9^{2x+5}$ d. $12^{3x-9} = 1$ e. $5^{x^2} = 125(5^{2x})$

Solution:

Express *each* side of the equation as a single power with the *same* base. Equate the exponents and solve.

a. $10^{x+4} = 1000^{x-4}$ b. $8^{x+2} = \left(\frac{1}{4}\right)^{x+3}$ c. $\left(\sqrt{3}\right)^{3x} = 9^{2x+5}$ d. $12^{3x-9} = 1$ e. $5^{x^2} = 125(5^{2x})$

Example 3: Solve Problems Involving Exponential Equations

A painting quadruples in value every 15 years. It is currently worth $1000.

a. Write an exponential function that models the value, V, of the painting after t years.
b. Use your equation to determine the value of the painting in 12 years.
c. Use your equation to determine the time needed for the painting to be worth $32 000.

Solution:

a. The exponential function can be written in the form $y = ac^{bx}$:

b. Substitute t = 12 into your equation and solve for V:

c. Substitute V = 32 000 into your equation and solve for t:

Example 4: Solve Problems Involving Exponential Equations

The half-life of a radioactive substance is 4 days.

a. Write an exponential function that models the amount, A, of the substance remaining after t days.
b. Use your equation to determine the time that must pass until there is 12.5% of the substance remaining.

Solution:

a. The exponential function can be written in the form $y = ac^{bx}$:

b. Substitute A = _____ into your equation and solve for t: