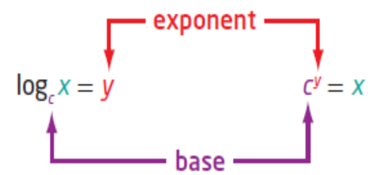
Understanding Logarithms

- The idea of logarithms is to *reverse* the operation of exponentiation (raising a number to an exponent). For example, we know that 2 cubed equals 8, or **$2^3 = 8$**. A logarithm is an exponent to which a fixed base must be raised to obtain a specific value. So, the logarithm of 8 with respect to base 2 equals 3, or **$\log_2 8 = 3$**.

- Equations in *logarithmic* form can be written in *exponential* form and vice versa.
  Logarithmic Form: $\log_c x = y$ **Exponential Form:** $c^y = x$
  Example: $\log_7 49 = 2$ **Example:** $7^2 = 49$



- The *inverse* of an exponential function $y = c^x, c > 0, c \neq 1$ is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the *inverse* of a logarithmic function $y = \log_c x, c > 0, c \neq 1$ is $x = \log_c y$ or, in exponential form $y = c^x$. The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line y=x.

- A ***common logarithm*** has base 10. Common logs are usually written without the base, that is, $\log_{10} x = \log x$.

- Another commonly used base is *e.* The number *e* (sometimes called Euler's number or Napier's constant) is an important mathematical constant. It is an irrational number, approximately equal to 2.71828. Logarithms with base *e* are known as ***natural logarithms***. The abbreviation "ln" is used to indicate the natural log and the base *e* is not included, that is, $\log_e x = \ln x$.

Example 1 : Graph the Inverse of an Exponential Function

Sketch the graph of the exponential function $y = 2^x$. State its inverse. Then, sketch the graph of the inverse function and identify the following characteristics of the inverse graph:

- domain and range
- x- and y- intercepts, if they exist
- the equations of any asymptotes

$y = 2^x$			$f^{-1}(x) = \log_2 x$	
x	y		x	y
−3				
−2				
−1				
0				
1				
2				
3				

Solution:

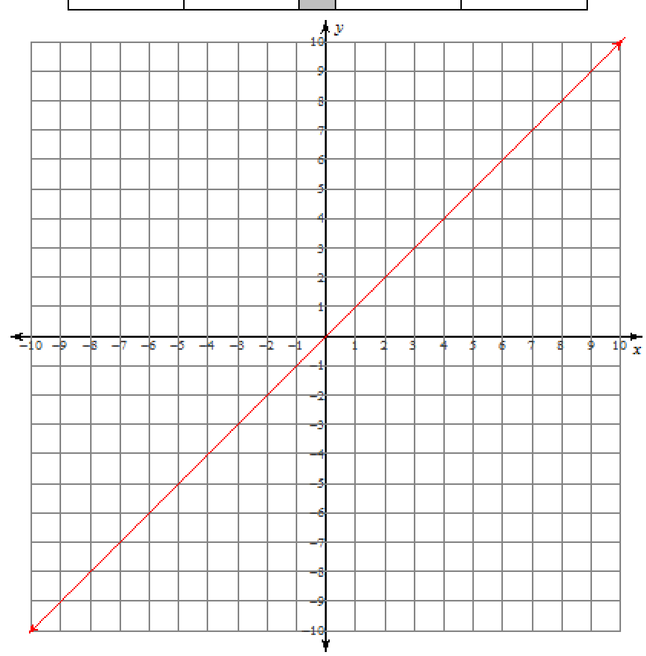
Complete the table of values for $y = 2^x$ and its inverse function.

Write the inverse of $y = 2^x$: ______________

In logarithmic form, the inverse function is:

____

Sketch the graph of $y = 2^x$ and its inverse on the same grid.



For the inverse (logarithmic) function identify:

domain: ____________________ range: ____________________

x-intercept: _____ y-intercept: __________

vertical asymptote: ______________

Example 2: Change the Form of an Expression

For each expression in exponential form, rewrite it in logarithmic form. For each expression in logarithmic form, rewrite it in exponential form.

a. $3^4 = 81 \leftrightarrow$ ________________ b. $64^{\frac{1}{2}} = 8 \leftrightarrow$ ________________

c. $\log_5 125 = 3 \leftrightarrow$ ____________ d. $\log 10\,000 = 4 \leftrightarrow$ ____________

Example 3: Evaluate a Logarithm

Evaluate the following logarithms.

a. $\log_6 216$ b. $\log_9 1$ c. $\log_2 \frac{1}{64}$ d. $\log_3 \sqrt{27}$

Example 4: Determine a Value in a Logarithmic Expression

Solve for x.

a. $\log_2 32 = x$ b. $\log_x \left(\frac{1}{27}\right) = -3$ c. $\log_4 x = -2$ d. $\log_x 32 = \frac{5}{3}$

Example 5: Estimate the Value of a Logarithm

Without using graphing technology, estimate to one decimal place the value of $\log_2 52$.

Solution:
Think: "What is the exponent that must be applied to base 2 to obtain 52?", and then use systematic trial.

$\log_2 52 =$ ________