Extra Trig Identity Questions

1. Given $\tan\theta = \frac{8}{15}$, find the values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$ without solving for $\theta, 180^{\circ} \le \theta \le 270°$.

2. If $\sin\alpha = \frac{4}{5}$ in Quadrant I and $\cos\beta = -\frac{12}{13}$ in Quadrant II, determine the exact value of $\cos(\beta - \alpha)$.

3. If $\sin\theta = -\frac{8}{9}$ (in Quadrant III), evaluate $\cos 2\theta$.

4. Prove the following:

   a) $\frac{\sin 2x}{1+\cos 2x} = \tan x$

   b) $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

   c) $\tan 2x - 2\tan 2x \sin^2 x = \sin 2x$

   d) $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

   e) $\cos(30^{\circ} + x) = \frac{\sqrt{3}\cos x - \sin x}{2}$

   f) $2\csc 2x \tan x = \sec^2 x$

   g) $\frac{\cos x}{\csc x} - \frac{\sin x}{\tan x} = \frac{\sin x - 1}{\sec x}$

   h) $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

   i) $\tan 2x \tan x + 2 = \frac{\tan 2x}{\tan x}$

   j) $\frac{\cos x + \sin x}{\cos x - \sin x} = \frac{1+\sin 2x}{\cos 2x}$

5. Solve the following for the indicated interval.

   a) $\tan 2x - \cot 2x = 0,\ -90^{\circ} \le x < 270^{\circ}$

   b) $\sin 2x \cos x - \cos 2x \sin x = 0, -\frac{\pi}{2} < x \le \frac{3\pi}{2}$

   c) $\sin 4x - \cos 2x = 0,\ 0 \le x < 180^{\circ}$

   d) $\cos 2x + 1 - \cos x = 0,\ 0 \le x \le 2\pi$

   e) $\sin 2x + \sin x = 6\cos x + 3,\ -180^{\circ} \le x \le 180^{\circ}$

   f) $3\tan x = \tan 2x,\ 0 \le x < \pi$

   g) $4\tan x - \sec^2 x = 0,\ 0 < x \le 360^{\circ}$

   h) $3\cos x - 4\sin x = 0,\ -180^{\circ} \le x < 180^{\circ}$

6. Solve for all possible values of x (in degrees)

   a) $\sin 2x - 1 = \cos 2x$

   b) $\cos 2x = \sin x$

   c) $\cos(45^{\circ} - x) = \sin(30^{\circ} + x)$

   d) $3\sin x = \cos(x + 60^{\circ})$

   e) $\tan x \tan 2x = 2$

   f) $\sin(x + 60^{\circ}) = \cos x$

   g) $9\sin x = \csc x$