**Introduction to Determining the Limit of a Function**

The limit of a function is defined as the range value (*y*-value) that a function approaches as *x* approaches a certain value. One way to determine this outcome is to graph the function and visually determine the range value that corresponds to the value *x* is approaching.

For example, a function is defined as *f* (*x*) = 2*x* + 1, and you are asked to determine the limit of 2*x* + 1 as *x* approaches 2. You begin by drawing the graph of the function like this:



Here, you can see that no matter the direction from which you approach 2 on the *x*-axis, the corresponding *y*-value approaches 5.

Now, observe the following table of values based on *y* = 2*x* + 1:

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| 1.5 | 4 |
| 1.9 | 4.8 |
| 1.99 | 4.98 |
| 1.999 | 4.998 |
| 1.9999 | 4.9998 |
| 1.99999 | 4.99998 |
| *x* | *f* (*x*) |
| 2.5 | 6 |
| 2.1 | 5.2 |
| 2.01 | 5.02 |
| 2.001 | 5.002 |
| 2.0001 | 5.0002 |
| 2.00001 | 5.00002 |

Notice that the closer *x* gets to the value of 2 from either direction, the closer *y* gets to the value of 5. You can summarize your findings by writing the limit of a function with the proper limit notation:



This reads as **the limit of the function *f* (*x*) as *x* approaches a is *L***. For the example above, you write the following:



As you have seen in previous lessons, some functions produce graphs with asymptotes, which are lines that the graph of a function approaches but never touches or crosses. Limits do not exist when approaching *x*-values that correspond to the location of vertical asymptotes, or if they do exist, they are not a finite value. This will be explained in more detail in subsequent lessons.

For now, consider the graph of the function :



There is a vertical asymptote at *x* = 3 because the function is undefined at *x* = 3 (substituting *x* = 3 into the function would make the denominator zero).

As *x* approaches 3 from either side, there is no finite number that *y* approaches. Therefore, the limit as *x* approaches 3 Does Not Exist (DNE).



You have also studied graphs that have a point of discontinuity. If the equation of the function  can be simplified to *y = x +* 3, *y* ≠ 2.

The graph is show below. There is a point of discontinuity at (2, 5):



On the graph, as *x* approaches 2, the *y*-value approaches 5. Even though the point (2, 5) is a point of discontinuity, the closer the *x*-value gets to 2, the closer the *y*-value will get to 5. Therefore, .