Introduction to Graphing Composite Functions

A **composition** of functions is substituting one function into another function. The range of the first function will become the domain of the second function:

* The composition of *f* (*x*) and *g* (*x*) is defined as *f* (*g* (*x*)) and is formed when the equation of *g* (*x*) is substituted into the equation of *f* (*x*).
* *f* (*g* (*x*)) is read as "*f* of *g* of *x*", meaning that the output of *g* (*x*) becomes the input for *f* (*x*).
* Another way to write *f* (*g* (*x*)) is (*f* ∘ *g*)(*x*). This is not to be confused with multiplication. (*f* ∘ *g*)(*x*) is not the same as (*f* ⋅ *g*)(*x*). (*f* ∘ *g*)(*x*) is also read as "*f* of *g* of *x*".

Example 1

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Express *f* (*g* (*x*)) and *g* (*f* (*x*)) as functions of *x*.

*f* (*x*) = 3*x* – 5          *g* (*x*) = *x*2 – *x*

To determine the equation of a composite function, substitute the second function into the first as read from left to right.

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| step1 | To compose *f* (*g* (*x*)), substitute the equation of *g* (*x*) into the equation of *f* (*x*):  *f* (*g* (*x*)) = 3 (*g* (*x*)) − 5             = 3(*x*2 − *x*) − 5             = 3*x*2 − 3*x* − 5 |
| step2 | To compose *g* (*f* (*x*)), substitute the equation of *f* (*x*) into the equation of *g* (*x*):  *g* (*f* (*x*)) = (*f* (*x*))2 − *f* (*x*)             = (3*x* − 5)2 − (3*x* − 5)             = 9*x*2 − 30*x* + 25 − 3*x* + 5             = 9*x*2 − 33*x* + 30 |

Two functions—*f* (*x*) and *g* (*x*)—can be combined using composition to produce two new functions—*f* (*g* (*x*)) and *g* (*f* (*x*)).

**Example 2**

To evaluate a composite function in the form of *f* (*g* (*x*)) at a specific value, you have two choices:

* determining the equation of the composite function first (as shown in Example 1) and then evaluating for the value of *x*.
* substituting the value into the equation for *g* (*x*) first and then substituting the result into *f* (*x*).

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| step1 | Using the functions *f* (*x*) = 4*x* + 1 and *g* (*x*) = 3 − *x*2, determining *f* (*g* (−1)):  *g* (−1) = 3 − (−1)2 = 3 − 1 = 2 |
| step2 | Now find *f* (2):  *f* (2) = 4 (2) + 1 = 9  Therefore, *f* (*g* (−1)) = 9. |

Note: Sometimes you will encounter a function within another function, and both functions are needed to answer a question or analyze a problem. The domain of the second function must connect to the first function. The notation *f*(*g*(𝑥)) is a composition of the two functions: *f* and *g*.