Introduction to Performing Operations on Functions

You have had exposure to functions and their graphs in the previous sections. In this lesson you will form new functions by adding, subtracting, multiplying, and dividing existing functions. This skill is used to solve problems by using tables, graphs, and equations.

Algebraic functions can be combined by adding, subtracting, multiplying, and dividing them.

**Adding Functions**

Function *f* (*x*) and *g* (*x*) can be added to give a third function *h* (*x*).

Example

If *f* (*x*) = 6*x* + 3 and *g* (*x*) = 2*x* + 5,
then *h* (*x*) = *f* (*x*) + *g* (*x*)
               = 6*x* + 3 + 2*x* + 5
               = 8*x* + 8

**Note**: *h* (*x*) = *f* (*x*) + *g* (*x*) can also be written as *h* (*x*) = (*f* + *g*)(*x*).

**Subtracting Functions**

Two functions can also be subtracted.

Example

If *f* (*x*) = 6*x* + 3 and *g* (*x*) = 2*x* + 5,
then *h* (*x*) = *f* (*x*) − *g* (*x*)
               = 6*x* + 3 − (2*x* + 5)
               = 4*x* − 2

**Note**: *h* (*x*) = *f* (*x*) − *g* (*x*) can also be written as *h* (*x*) = (*f* − *g*)(*x*).

**Multiplying Functions**

Algebraic functions can be combined by multiplying. For example, function *f* (*x*) and *g* (*x*) can be multiplied to give a third function *h* (*x*).

Example

If *f* (*x*) = *x* + 2 and *g* (*x*) = 2*x* − 1,
then *h* (*x*) = *f* (*x*) × *g* (*x*)
               = (*x* + 2)(2*x* − 1)
               = 2*x*2 + 3*x* − 2

**Note**: *h* (*x*) = *f* (*x*) × *g* (*x*) can also be written as *h* (*x*) = (*f* · *g*)(*x*).

Notice that the product of 2 non-constant, linear functions is a quadratic function. If you graph all 3 functions on the same grid, you would notice that the *x*-intercept of each linear function would match the *x*-intercepts of the quadratic function.

**Dividing Functions**

Algebraic functions can also be divided:

Example

If *f* (*x*) = *x* + 2 and *g* (*x*) = 2*x* − 1,
then  in which 

**Note**:  can also be written as , in which *g* (*x*) ≠ 0.

The range of a product or quotient combination is best determined by using its graph.

**Determining the Domain**

The domain of the combined function formed by the sum or difference of two functions is the domain common to the individual functions. In other words, the domain of the combined function is the overlap between the domains of the original two functions.

Example

Domain of *f* (*x*): {*x* | *x* ≤ 6, *x* ∈ *R*}
Domain of *g* (*x*): {*x* | *x* ≥ −2, *x* ∈ *R*}
Domain of *h* (*x*): {*x* | −2 ≤ *x* ≤ 6, *x* ∈ *R*}

**Determining the Range**

The range is the sum or difference of the range values of the original graph and would depend on the domain. The range is best determined by using a graph.