15.3 Resistance to Flow of Charge

SECTION OUTCOMES

- Apply Ohm's law to circuits.
- Describe the concepts and units related to electricity and magnetism in terms of electron flow.
- Design and conduct an experiment to investigate major variables relating electric potential, current, and resistance.

KEY TERMS

- resistivity
- unit of resistance (ohm)
- non-linear or non-ohmic resistance

Everything that moves experiences frictional forces that resist that motion. The energy used to overcome frictional resistance within mechanical and electrical systems, such as automobile engines and electric motors, costs billions of dollars each year. Like every other system, the frictional effects in conductors offer resistance to the current passing through them. Similar to mechanical systems, friction in electric conductors produces thermal energy that is radiated away from the conductor. This energy is no longer available to do work. Electrical engineers' knowledge of the factors that affect resistance enable them to design systems that reduce energy loss to a minimum.

Figure 15.14 Overcoming resistance is an important aspect of electric-circuit design.

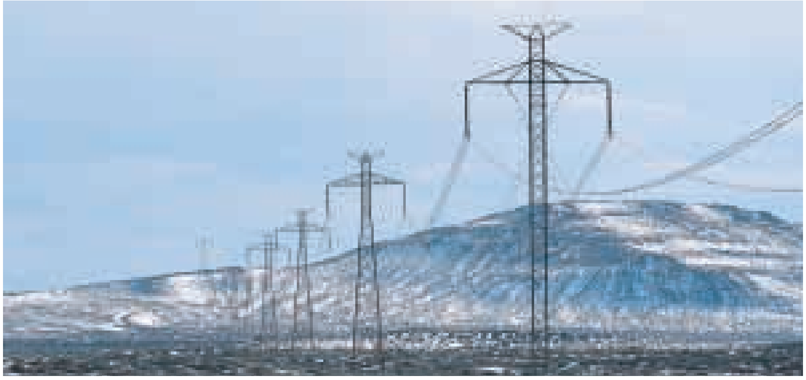
Factors Affecting the Resistance of a Conductor

You can compare a metal conductor carrying electric current to a water pipe carrying water current. For a water pipe of a fixed diameter, the longer the pipe, the greater the drag it exerts on the water passing through it. Similarly, for an electric conductor with a fixed diameter, resistance increases proportionately with the *length* ($R \propto L$). Thus, a 2 m length of a particular conductor has twice as much resistance as a 1 m length of the same conductor.

For two conductors of equal length, changing the cross-sectional area changes the resistance. Again, for a water pipe, the bigger the cross-sectional area of the pipe, the lower the drag on the water moving inside it. It is the cross-sectional area that provides the space in which the current travels. Therefore, doubling the cross-sectional area doubles the space for the current to move and halves the resistance. For electric conductors, resistance varies inversely as the *cross-sectional area* ($R \propto 1/A$). For very long extension cords, the resistance due to the increased length of the cord can cause a significant energy loss. To lower the resistance, long conductors are made of thicker wire, which increases cross-sectional area and thus reduces resistance. Conversely, light bulb filaments must have a large resistance so that the energy will be transformed into light and thermal energy. To increase resistance, filaments are made very thin.

Conceptual Problems

- Consider the different electric cords typically found around the home, such as the cords on an iron, lamp, television set, small space heater, and toaster; a standard extension cord; and the cords for plugging in a vacuum cleaner or a car's block heater. Think about the length and thickness of each cord. Explain why each cord has its own specific size. For example, why is a toaster cord shorter and thicker than a lamp cord?
- Electric energy is often transmitted over many kilometres of conductors from remote generating stations to communities as shown in the photo. What characteristics should electrical engineers design into these conductors? Do research to find out how the losses to heat in the transmission lines are kept to a minimum.



If you combine the relationship of the resistance of a conductor to its length and cross-sectional area, the result is $R \propto \frac{L}{A}$. Any proportionality can be written as an equality if a proportionality constant is included. In the case of resistance, the symbol used for this proportionality constant is the Greek letter *rho* (ρ). The equation for the resistance of a conductor can now be written: $R = \rho\frac{L}{A}$. The value of the proportionality constant, ρ, is called the **resistivity** and is a property of the material from which the conductor is made.

PHYSICS FILE

The thickness of wire is called its "gauge." As the gauge of a wire increases, the wire becomes thinner. When electricians wire the circuits of a house, they usually use 14 gauge wire, while the lighter wire in the cord of a small appliance might be 18 gauge wire.

Diameters/Resistances of Some Gauges of Copper Wire

Gauge	Diameter (mm)	Resistance ($\times 10^{-3}\Omega$/m)
0	9.35	0.31
10	2.59	2.20
14	1.63	8.54
18	1.02	21.90
22	0.64	51.70

RESISTANCE OF A CONDUCTOR

The resistance of a conductor is the product of the resistivity and the length divided by the cross-sectional area.

$$R = \rho\frac{L}{A}$$

Quantity	Symbol	SI unit
resistance	R	Ω (ohm)
resistivity	ρ	Ω · m (ohm metres)
length of conductor	L	m (metres)
cross-sectional area	A	m^2 (square metres)

Unit Analysis

$$(\text{ohm metres})\ \frac{\text{metres}}{\text{square metres}} = \Omega \cdot \cancel{m}\frac{\cancel{m}}{\cancel{m^2}} = \Omega$$

The resistance of a conductor with a particular length and cross-sectional area depends on the *material* from which it is made. At room temperature, copper is one of the best conducting (lowest resistance) metals. Table 15.1 includes resistivity values for carbon and germanium, which are semiconductors, and for glass, which is an insulator. Insulators are sometimes thought of as conductors with extremely high resistances. By examining Table 15.1, you can see that glass has about 10^{18} to 10^{22} times the resistance of copper.

Table 15.1 Resistivity of Some Conductor Materials

Material	*Resistivity, ρ ($\Omega \cdot m$)
silver	1.6×10^{-8}
copper	1.7×10^{-8}
aluminum	2.7×10^{-8}
tungsten	5.6×10^{-8}
Nichrome™	100×10^{-8}
carbon	3500×10^{-8}
germanium	0.46
glass	10^{10} to 10^{14}

*Values given for a temperature of 20°C

Finally, the *temperature* of the conductor affects the resistance. The electrons that move inside a metallic conductor are the electrons from the outermost orbit of the atoms of the metal. Thus, they are the electrons that are most loosely held by the atoms of the metal. These outermost electrons of good conductors can move quite freely within the metal, behaving much like the molecules of a gas. As you heat the metal, these electrons begin to move more randomly at higher speeds inside the metal. As a result, it is more difficult to organize them into a current. Near 20°C, copper increases its resistance by about 0.39% for each degree of temperature increase. Conversely, lowering the temperature reduces the resistance.

MODEL PROBLEM

Using Resistivity

Calculate the resistance of a 15 m length of copper wire, at 20°C, that has a diameter of 0.050 cm.

Frame the Problem

- The *electric resistance* of a conductor depends on its *length*, *cross-sectional area*, the *resistivity* of the conducting material, and the *temperature*.
- These variables are related by the equation for resistance of a conductor.
- The *resistivity* of copper at 20°C is listed in Table 15.1.

Identify the Goal

Resistance, R, of the copper conductor

Variables and Constants

Known	**Implied**	**Unknown**
$d = 0.050$ cm	$\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$	R
$L = 15$ m		A

Strategy

Calculations

Use the equation relating resistance to resistivity and dimensions of the conductor.

$$R = \rho \frac{L}{A}$$

Convert diameter to SI units. (All others are in SI units.)

$$0.050\ \cancel{\text{cm}}\ \frac{\text{m}}{100\ \cancel{\text{cm}}} = 5.0 \times 10^{-4}\ \text{m}$$

Find the cross-sectional area from the diameter.

$$A = \pi r^2$$

$$r = \frac{d}{2}$$

$$r = \frac{5.0 \times 10^{-4}\ \text{m}}{2} = 2.5 \times 10^{-4}\ \text{m}$$

$$A = \pi(2.5 \times 10^{-4}\ \text{m})^2$$

$$A = 1.96 \times 10^{-7}\ \text{m}^2$$

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Strategy

The values are all known, so substitute into the equation for resistance.

Calculations

$$R = (1.7 \times 10^{-8}\ \Omega \cdot \text{m})\ \frac{15\ \text{m}}{1.96 \times 10^{-7}\ \text{m}^2}$$

$$R = 1.3\ \frac{\Omega \cdot \cancel{\text{m}} \cdot \cancel{\text{m}}}{\cancel{\text{m}^2}}$$

$$R = 1.3\ \Omega$$

The conductor has a resistance of 1.3 Ω.

Validate

The units cancel to give ohms, which is correct for resistance. At first glance, a resistance of 1.3 Ω seems large for a 15 m length of copper given that copper is a very good conductor. However, the wire is very fine, only 0.5 mm in diameter, giving a cross-sectional area of only $1.96 \times 10^{-7}\ \text{m}^2$. This small area accounts for the resistance.

PRACTICE PROBLEMS

Use the data provided in Table 15.1 and the table in the Physics File on page 706 to solve the following problems.

16. What is the resistance of 250 m of aluminum wire that has a diameter of 2.0 mm?

17. What is the length of 18 gauge Nichrome™ wire that has a resistance of 5.00 Ω?

18. The resistance of a 100 W (1.00×10^2 W) light bulb is 144 Ω. If its tungsten filament is 2.0 cm long, what is its radius? (The bulb is not turned on.)

19. An extension cord is made of 14 gauge aluminum wire. Calculate the resistance of this cord if its length is 35 m.

20. A square carbon rod is 24 m long. If its resistance is 140 Ω, what is its width?

INVESTIGATION 15-B

Current, Resistance, and Potential Difference

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

If you change one property in a circuit, such as the potential difference or the resistance, you would expect that another characteristic would change in response. Does it? If so, how does it change?

Problems

- How is current affected by a change in the potential difference if the resistance remains constant?
- How is current affected by a change in the resistance if the potential difference remains constant?

Hypotheses

Formulate hypotheses for the relationships between potential difference and current in a circuit, and between resistance and current in a circuit.

Equipment

- variable DC power supply
- multi-range ammeter
- multi-range voltmeter
- Nichrome™ wire (1 m, approximately 22 gauge)
- metre stick
- insulated connecting leads with alligator clips
- thumbtacks

CAUTION If the current in the Nichrome™ wire is large, the wire will become very hot. When your circuit has more than 1 A of current, you could be burned if you touch the wire.

CAUTION When wiring your circuit, be sure to connect the conductor to the anode last. Making this connection last will ensure that you do not accidentally create a live circuit while you are making other connections.

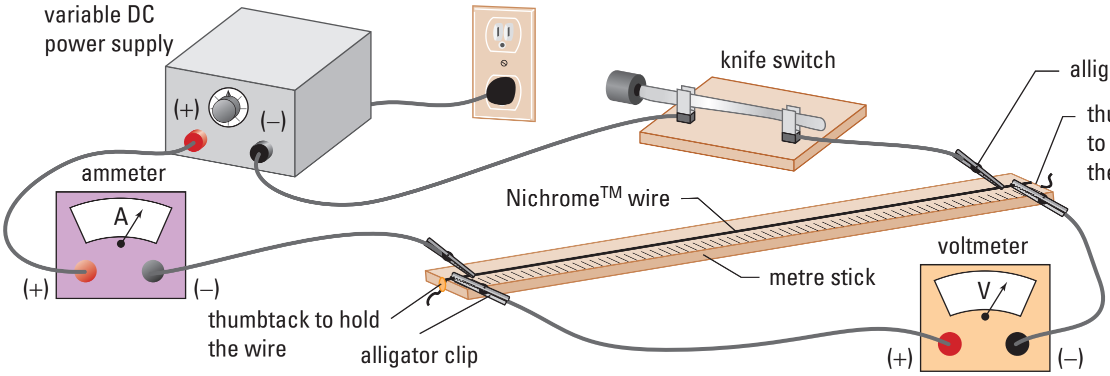
CAUTION DC ammeters and DC voltmeters must be connected properly to avoid damaging them and to ensure that they make proper measurements for the desired parts of the circuit. To avoid damaging the meters, make sure that they are connected in the correct direction. Like DC power supplies, DC meters have an anode (red or positive post) and a cathode (black or negative post). The meter must always be connected so that the current is moving from the red post to the black post (downhill) as it passes through the meter.

Procedure

Part 1

Connecting the Circuit

1. Study the figure to see how all connections will be made. Then, follow the order of making connections in the following steps.



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2. Connect the cathode (black post) of the power supply to one end of the load. (In the first part of the investigation, the load will be about 50 cm of the Nichrome™ wire.)
3. Connect the other end of the load to the black post of the ammeter.
4. Connect the red post of the ammeter to the anode of the power supply. Do *not* connect the voltmeter until these connections are complete and have been checked.
5. Before turning on the power supply, be sure that the knob is turned completely down (counterclockwise). Turn on the power supply and increase the potential difference very slightly. Check the ammeter to see if the current is in the correct direction. If everything is correct, turn off the power supply or disconnect the wire at the anode.
6. Connect the voltmeter across the load (Nichrome™ wire). Be sure that the red post of the voltmeter is connected to the end of the load that is closest to the anode (red post) of the power supply. Reconnect the power supply and increase the potential difference of the power supply slightly. Check that the needle of the voltmeter is moving in the correct direction.

Part 2

Current versus Potential Difference

7. Choose about 50 cm of the Nichrome™ wire as the load. Complete the circuit as described above, checking to see that the ammeter and the voltmeter are connected properly. If the voltmeter and the ammeter are multi-range meters, be sure to start on the least-sensitive range (the highest possible voltage or current readings) and move to more-sensitive ranges as conditions permit. If you are unsure of how to read the meter, consult your teacher.
8. Make a data table with the column headings: Trial, Potential Difference (V), Current (I).
9. With all connections in place, set the power supply at a low value. Read and record the current through, and the potential difference across, the load.
10. Keep the resistance constant; that is, do not move the alligator clips on the Nichrome™ wire. Increase the potential difference of the power supply slightly and, again, read the current and potential difference.
11. Repeat step 10 several times, until you have five or six readings.

Part 3

Current versus Resistance

CAUTION Remember that a large current will cause excessive heating.

12. Use the same circuit connections as in Part 1.
13. Choose a length of Nichrome™ wire (about 12 to 15 cm) as your standard resistance. The resistance of this length of wire will be considered one unit of resistance. (**Note:** Since you have created this standard resistance, you get to name the unit.)
14. Make a data table with the column headings: Trial, Resistance (in the units you gave it), Current. Leave two more columns for data to be used when you make your analysis.
15. Start with one unit of your standard resistance as a load. Increase the potential difference of the power supply until the ammeter registers a current of about 0.5 A. Record this potential difference. In your table, record the current and resistance as Trial 1.

16. Set the resistance of the load to be one unit larger than the previous trial ($R \propto L$). Check the potential difference across the load and reset it to the original value. Read and record the new values of current and resistance.

17. Repeat step 16 until you have four or five separate trials.

Analyze and Conclude

1. Graph your data for current (I) versus potential difference (V). Plot I as a function of V.

2. What does the shape of the line on your graph indicate about the relationship between potential difference and current when resistance is constant? How do these results compare with your hypothesis?

3. Write a summary statement describing your conclusion about the relationship of current to potential difference.

4. Graph your data for current (I) versus resistance (R). Plot I as a function of R.

5. What does the shape of the graph suggest is the probable relationship between the variables, R and I?

6. Based on your interpretation of the graph, decide how to mathematically modify the variable R so you could plot the adjusted value as a function of I and obtain a straight line. (Hint: Mathematical modifications could be such things as squaring, taking the square root, inverting, or multiplying by a constant or by the other variable.)

7. Calculate values for the adjusted independent variable. In one of the empty columns, write your calculated values.

8. Graph the current as a function of the newly created variable.

9. Was your decision in question 6 correct? If not, continue this process until you have found a modified form of R that gives a straight line.

10. Was your original hypothesis about the relationship between current and resistance correct? If not, what is the correct relationship?

11. Write a summary statement about the relationship between current and resistance when the potential difference is held constant.

Apply and Extend

12. Combine these two relationships into one. Write your results in the form of a proportionality. (**Note:** This is the basis of what is known as Ohm's law.)

13. Convert your relationship into an equation by including a constant (k) of proportionality.

14. Calculate the value of your constant by using the data from one of the trials in the second part of the investigation. Rewrite your relationship with your value for k included. This is Ohm's law for your standard resistance.

15. The value you find for k for your apparatus depends on the length of Nichrome™ wire that you arbitrarily chose as your standard resistance. Calculate the length of wire segment that you would have to use as your standard resistance if you wanted the value for k to equal one. By definition, the length of Nichrome™ wire that would produce a proportionality constant equal to one ($k = 1$) has a resistance of one ohm.

Ohm's Law

In 1826, German physicist Georg Simon Ohm conducted the original experiments in resistance in electric circuits, using many lengths and thicknesses of wire. He studied the current passing through the wire when a known potential difference was applied across it. From his data, Ohm developed the mathematical relationship that now bears his name, Ohm's law.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, go to the Internet site above for several laboratory activities.

OHM'S LAW

The potential difference across a load equals the product of the current through the load and the resistance of the load.

$$V = IR$$

Quantity	Symbol	SI unit
potential difference	V	V (volt)
current	I	A (ampere)
resistance	R	Ω (ohm)

Unit Analysis

(potential difference) = (current)(resistance) = A · Ω = V

Note: One ampere times one ohm is equivalent to one volt.

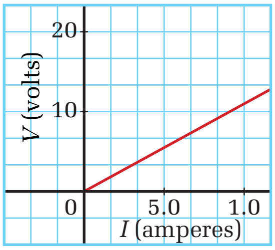
The **unit of resistance**, the **ohm**, is defined in accordance with Ohm's law. One ohm is defined as the amount of electric resistance that will allow one ampere of current to move through the resistor when a potential difference of one volt is applied across the resistor.

$$\left(1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}}\right)$$

Initially, Ohm's law seemed to be the answer to the problem of defining how a load would affect current. Unfortunately, its usefulness is limited to metal conductors at stable temperatures. For the majority of loads, such as motors, electronic capacitors, and semiconductors, the resistance changes with a change in the potential difference. Even a light bulb does not obey Ohm's law, because the heating of the filament causes its resistance to increase. When a load does not obey Ohm's law, the graph of I versus V is not a straight line. Devices and materials that do not obey Ohm's law are said to be **non-linear** or **non-ohmic**. Metallic conductors are, however, a sufficiently important and large class of materials that demonstrate the law is still extremely useful.

Conceptual Problems

- Consider the graph of potential difference versus current such as shown here. What is the significance of the slope of the line? (Hint: The equation for any straight line is $y = mx + b$, where m is the slope of the line and b is the y-intercept.)



MODEL PROBLEM

Applying Ohm's Law

What is the resistance of a load if a battery with a 9.0 V potential difference causes a current of 0.45 A to pass through the load?

Frame the Problem

- A battery creates a *potential difference* that provides energy to cause a *current* to flow in a circuit.
- A *load* resists the flow of current.
- You can find the *resistance* of the load by using Ohm's law.

Identify the Goal

The resistance, R, of the load

Variables and Constants

Known	**Unknown**
$V = 9.0\ \text{V}$	R
$I = 0.45\ \text{A}$	

Strategy

Apply Ohm's law.

Calculations

$$V = IR$$

Substitute first

$$9.0\ \text{V} = (0.45\ \text{A})\,R$$

$$\frac{9.0\ \text{V}}{0.45\ \text{A}} = \frac{\cancel{(0.45\ \text{A})}\,R}{\cancel{0.45\ \text{A}}}$$

$$R = 20\ \Omega$$

Solve for *R* first

$$\frac{V}{I} = \frac{\cancel{I}R}{\cancel{I}}$$

$$\frac{9.0\ \text{V}}{0.45\ \text{A}} = R$$

$$R = 20\ \Omega$$

If a 9.0 V potential difference across a resistance results in a current of 0.45 A, the resistance is $2.0 \times 10^1\ \Omega$.

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Validate

The data fit Ohm's law. The number 9 divided by approximately $\frac{1}{2}$ is the same as $9 \times 2 = 18$. The final answer, 20, is close to 18.

PRACTICE PROBLEMS

21. The heating element of an electric kettle draws 7.5 A when connected to a 120 V power supply. What is the resistance of the element?

22. A toaster is designed to operate on a 120 V (1.20×10^2 V) system. If the resistance of the toaster element is 9.60 Ω, what current does it draw?

23. A small, decorative light bulb has a resistance of 36 Ω. If the bulb draws 140 mA, what is its operating potential difference? (**Note:** The prefix "m" before a unit always means "milli-" or one one-thousandth. 1 mA is 1×10^{-3} A.)

24. The light bulb in the tail-light of an automobile with a 12 V electrical system has a resistance of 5.8 Ω. The bulb is left on for 8.0 min.

- **(a)** What quantity of charge passes through the bulb?
- **(b)** What was the current in the tail-light?

25. An iron transforms 3.35×10^5 J of electric energy to thermal energy in the 4.50 min it takes to press a pair of slacks. If the iron operates at 120 V (1.20×10^2 V), what is its resistance?

26. In Europe, some countries use 240 V (2.40×10^2 V) power supplies. How long will it take an electric kettle that has a resistance of 60.0 Ω to produce 4.32×10^5 J of thermal energy?

15.3 Section Review

1. I By what factor would the resistance of two copper wires differ if the second wire:

- **(a)** was double the length of the first?
- **(b)** was triple the cross-sectional area of the first?
- **(c)** had a radius that was half the radius of the first?
- **(d)** was half as long and twice as thick (twice the diameter) as the first?
- **(e)** was three times as long and a third the cross-sectional area of the first?

2. I What happens to the resistance of a conductor when the temperature of the conductor increases?

3. I Design an experiment that you would carry out to determine whether a particular circuit element was ohmic or non-ohmic. Explain how you would interpret the results.

UNIT PROJECT PREP

Your electric motor will require current to operate.

- What factors affect the current flow within a conductor?
- How can you increase or decrease the current flow within a motor?