15.4 Series and Parallel Circuits

Figure 13.15 Skiers arriving at the top of a ski lift have several different runs down the hill available to them. The route down the hill is a complex circuit of series and parallel runs, taking skiers back to the bottom of the lift.

SECTION OUTCOMES

- Apply Ohm's law to series and parallel circuits.
- Demonstrate understanding of the physical quantities of electricity.
- Synthesize information to solve electric energy problems.

KEY TERMS

- series
- parallel
- equivalent resistance
- internal resistance
- electromotive force
- terminal voltage

An extremely simple electric device, such as a flashlight, might have a circuit with one power source (a battery), one load (a light bulb), and one switch. In nearly every practical circuit, however, the power source supplies energy to many different loads. In these practical circuits, the loads may be connected in **series** (Figure 15.16) or in **parallel** (Figure 15.17). The techniques used to analyze these complex circuits are very similar to those you used to analyze simple circuits.

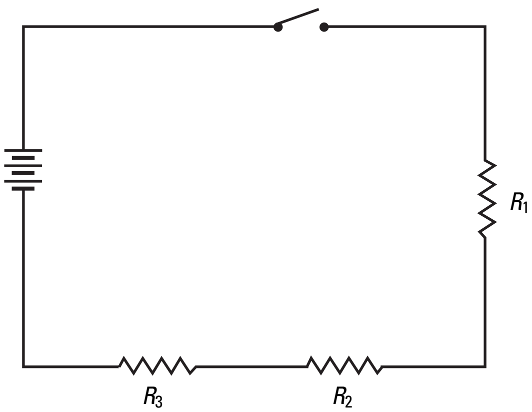

Figure 15.16 A circuit with resistances in series has only one closed path.

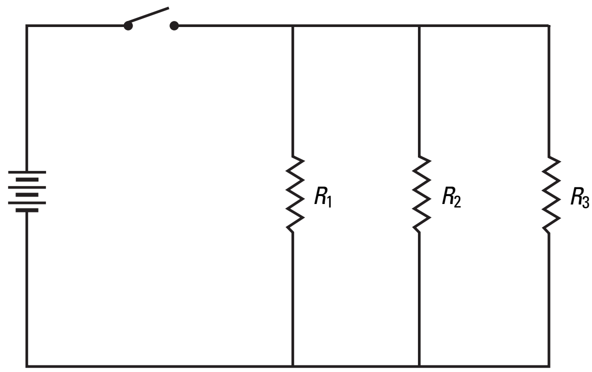

Figure 15.17 A circuit with resistances in parallel has several closed paths.

Series Circuits

A ski hill consisting of three downhill runs, one after the other, with level paths connecting the runs, is an excellent analogy for a series circuit. Since there is only one route down, the number of skiers going down each run would have to be the same. The total height of the three runs would have to equal the total height of the hill, as illustrated in Figure 15.18.

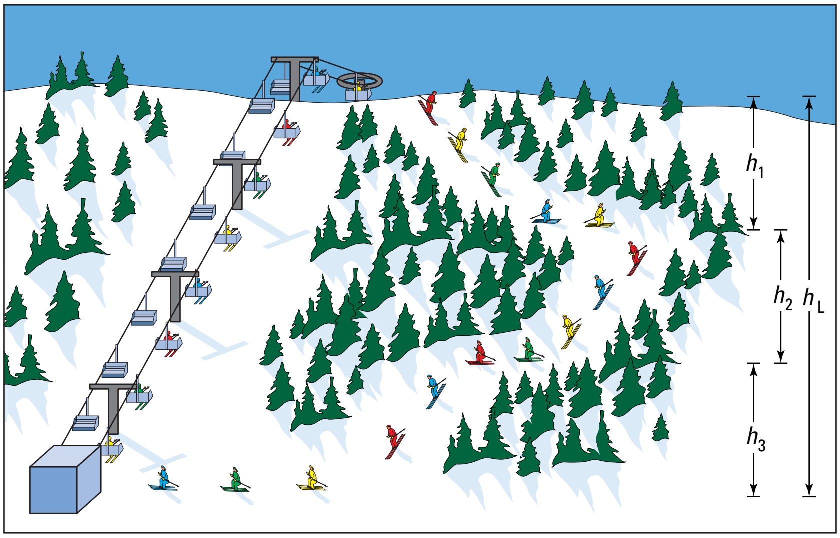

Figure 15.18 As skiers go around a ski circuit, the lift raises them to the top of the hill. Since the ski runs are in series, all of the skiers must ski down each of the runs, so that the number of skiers completing each run is the same. Each of the runs takes the skiers down a portion of the total height given to them by the ski lift. The combined height of the three runs must equal the height of the hill ($h_L = h_1 + h_2 + h_3$).

PHYSICS FILE

Just as skiers' gravitational potential energy drops as they ski down a hill, the electric potential energy of charges moving in a circuit drops when they pass through a load. Physicists often call the potential difference across a load a "potential drop." When the charges are given more potential energy in a battery or power source, they experience a "potential gain."

A series circuit consists of loads (resistances) connected in series, as was shown in Figure 15.16. The current that leaves the battery has only one path to follow. Just as the skiers in the previous analogy must all ski down each run in sequence, all of the current that leaves the battery must pass through each of the loads. An ammeter could be connected at any point in the circuit and each reading would be the same.

Also, just as the total height of the hill must be shared over the three runs, the potential difference of the battery must be shared over all three loads. Thus, a portion of the electric potential of the battery must be used to push the current through each load. If each load had a voltmeter connected across it, the total of the potential differences across the individual loads must equal the potential difference across the battery.

To find the **equivalent resistance** of a series circuit with *N* resistors, as illustrated in Figure 15.19 on the following page, analyze the properties of the circuit and apply Ohm's law.

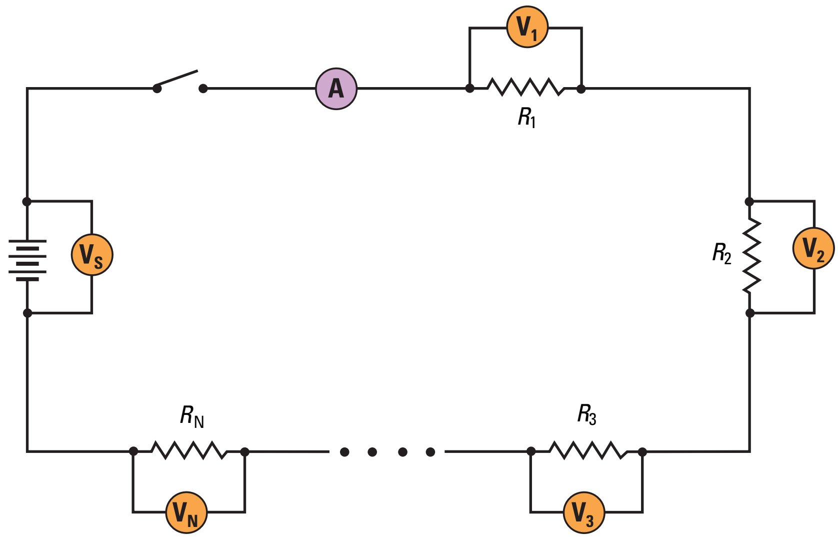

Figure 15.19 A circuit might consist of any number of loads. If this circuit had eight loads, R_N would represent R_8, and eight loads would be connected in series.

- Write, in mathematical form, the statement, "The total of the potential differences across the individual loads must equal the potential difference across the battery (source)." $$V_S = V_1 + V_2 + V_3 + \cdots + V_N$$
- By Ohm's law, the total potential difference across the battery must be equal to the product of the current through the battery and the equivalent resistance of the circuit. $$V_S = I_S R_{eq}$$
- For each load, the potential difference across that load must equal the product of the current through the load and its resistance. $$V_1 = I_1 R_1,\ V_2 = I_2 R_2,\ V_3 = I_3 R_3, \cdots, V_N = I_N R_N$$
- Substitute these expressions into the first expression for potential difference. $$I_S R_{eq} = I_1 R_1 + I_2 R_2 + I_3 R_3 + \cdots + I_N R_N$$
- Write, in mathematical form, the statement, "All of the current that leaves the battery must pass through each of the loads." $$I_1 = I_2 = I_3 = \cdots = I_N = I_S$$
- In the expression above, replace the symbol for each separate current with I_S, the current through the source. $$I_S R_{eq} = I_S R_1 + I_S R_2 + I_S R_3 + \cdots + I_S R_N$$
- Factor I_S out of the right side. $$I_S R_{eq} = I_S (R_1 + R_2 + R_3 + \cdots + R_N)$$
- Divide both sides by I_S. $$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

EQUIVALENT RESISTANCE OF LOADS IN SERIES

The equivalent resistance of loads in series is the sum of the resistances of the individual loads.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

Quantity	Symbol	SI unit
equivalent resistance	R_{eq}	Ω (ohm)
resistance of individual loads	$R_{1,2,3,\cdots N}$	Ω (ohm)

MODEL PROBLEM

Resistances in Series

Four loads (3.0 Ω, 5.0 Ω, 7.0 Ω, and 9.0 Ω) are connected in series to a 12 V battery. Find

(a) the equivalent resistance of the circuit

(b) the total current in the circuit

(c) the potential difference across the 7.0 Ω load

Frame the Problem

- Since all of the resistors are in series, the formula for the equivalent resistance for a series circuit applies to the problem.
- All of the loads are in series; thus, the current is the same at all points in the circuit. The current can be found by using the potential difference across the battery, the equivalent resistance, and Ohm's law.
- Ohm's law applies to each individual circuit element.

Identify the Goal

The equivalent resistance, R_{eq}, for the series circuit, the current, I, and the potential difference, V_3, across the 7.0 Ω resistor

Variables and Constants

Known	Unknown
$R_1 = 3.0\ \Omega$	R_{eq}
$R_2 = 5.0\ \Omega$	I
$R_3 = 7.0\ \Omega$	V_3
$R_4 = 9.0\ \Omega$	
$V_S = 12\ \text{V}$	

Strategy

Use the equation for the equivalent resistance of a series circuit.

Calculations

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$
$$= 3.0\ \Omega + 5.0\ \Omega + 7.0\ \Omega + 9.0\ \Omega$$
$$= 24\ \Omega$$

(a) The equivalent resistance for the four resistors in series is 24 Ω.

Write Ohm's law, in terms of current, and the equivalent resistance. Solve for the current in the circuit.

$1\ \frac{V}{\Omega}$ is equivalent to 1A.

$$V_S = I_S R_{eq}$$
$$I_S = \frac{V_S}{R_{eq}}$$
$$I_S = \frac{12\ V}{24\ \Omega}$$
$$I_S = 0.50\ A$$

(b) The current in the circuit is 0.50 A.

Use Ohm's law, the current, and the resistance of a single resistor to find the potential drop across that resistor.

Since the circuit has only one closed loop, the current is the same everywhere in the circuit, so $I_3 = I_S$.

1 A · Ω is equivalent to 1 V.

$$V_3 = I_3 R_3$$
$$I_3 = I_S$$
$$V_3 = (0.50\ A)(7.0\ \Omega)$$
$$V_3 = 3.5\ A \cdot \Omega$$
$$V_3 = 3.5\ V$$

(c) The potential drop across the 7.0 Ω resistor is 3.5 V.

Validate

If you now find the potential difference across the other loads, the sum of the potential differences for the four loads should equal 12 V.

$V_1 = (0.50\ A)(3.0\ \Omega)$ $\quad V_2 = (0.50\ A)(5.0\ \Omega)$ $\quad V_4 = (0.50\ A)(9.0\ \Omega)$

$V_1 = 1.5\ V$ $\quad V_2 = 2.5\ V$ $\quad V_4 = 4.5\ V$

$1.5\ V + 2.5\ V + 3.5\ V + 4.5\ V = 12\ V$

PRACTICE PROBLEMS

27. Three loads, connected in series to a battery, have resistances of 15.0 Ω, 24.0 Ω, and 36.0 Ω. If the current through the first load is 2.2A, calculate

(a) the potential difference across each of the loads

(b) the equivalent resistance for the three loads

(c) the potential difference of the battery

28. Two loads, 25.0 Ω and 35.0 Ω, are connected in series. If the potential difference across the 25.0 Ω load is 65.0 V, calculate

(a) the potential difference across the 35.0 Ω load

(b) the potential difference of the battery

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29. Two loads in series are connected to a 75.0 V battery. One of the loads is known to have a resistance of 48.0 Ω. You measure the potential difference across the 48.0 Ω load and find it is 40.0 V. Calculate the resistance of the second load.

30. Two loads, R_1 and R_2, are connected in series to a battery. The potential difference across R_1 is 56.0 V. The current measured at R_2 is 7.00 A. If R_2 is known to be 24.0 Ω, find

(a) the resistance of R_1

(b) the potential difference of the battery

(c) the equivalent resistance of the circuit

31. A 240 V (2.40×10^2 V) power supply is connected to three loads in series. The current in the circuit is measured to be 1.50 A. The resistance of the first load is 42.0 Ω and the potential difference across the second load is 111 V. Calculate the resistance of the third load.

Resistors in Parallel

The ski hill in Figure 15.20 provides a model for a circuit consisting of a battery and three resistors connected in parallel. The ski lift is, of course, analogous to the battery and the runs represent the resistors. Notice that the hill has three runs beside each other (in parallel) that go all of the way from the top to the bottom of the hill. The height of each of the runs must be equal to the height that the skiers gain by riding the lift up the hill ($h_L = h_1 = h_2 = h_3$).

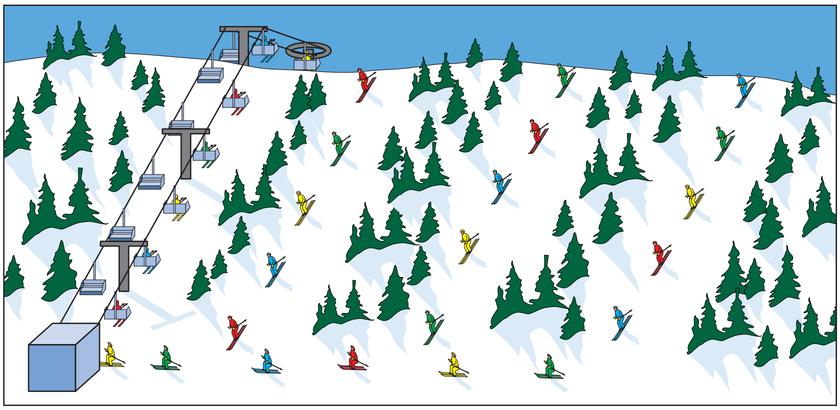


Figure 15.20 When skiers are on a hill where there are several ski runs that all are the same height as the hill, the runs are said to be *parallel* to each other.

The gravitational potential difference of the hill is analogous to the electric potential difference across a battery and resistors connected in parallel in a circuit. Similar to the height of the hill, the potential difference across each of the individual loads in a parallel circuit must be the same as the total potential difference across the battery (V_S). For example, in Figure 15.21, with N resistors in parallel, the mathematical relationship can be written as follows.

$$V_S = V_1 = V_2 = V_3 = \cdots = V_N$$

In the ski-hill analogy, the skiers themselves represent the current. A skier might select any one of the three runs, but can ski down only one of them. Since the skiers riding up the lift can go down only one of the hills, the sum of the skiers going down all three hills must be equal to the number of skiers leaving the lift.

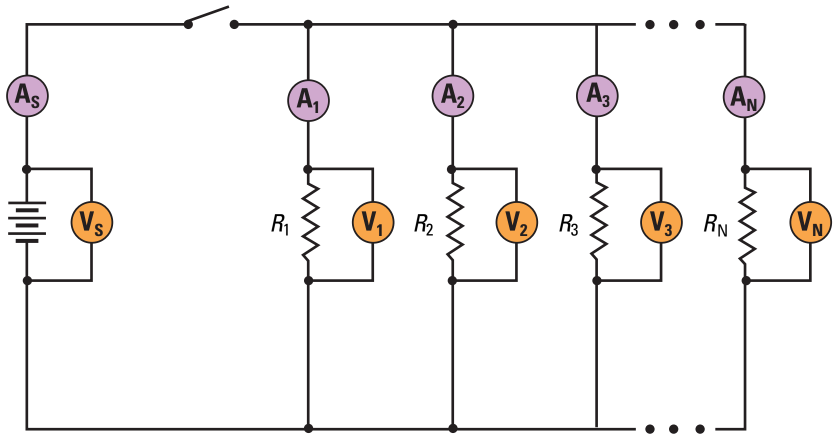


Figure 15.21 The *N* loads in this circuit are all connected in parallel with each other. The dots indicate where any number of additional loads could be connected in parallel with those present.

When the current leaving the battery (I_S) comes to a point in the circuit where the path splits into two or more paths, the current must split so that a portion of it follows each path. After passing through the loads, the currents combine before returning to the battery. The sum of the currents in parallel paths must equal the current entering and leaving the battery.

Knowing the current and potential difference relationships in a parallel circuit, you can use Ohm's law to develop an equation for the equivalent resistance of resistors in a parallel connection.

- Write, in mathematical form, the statement, "The sum of the currents in parallel paths must equal the current through the source."

$$I_S = I_1 + I_2 + I_3 + \cdots + I_N$$

- Write Ohm's law and solve $V = IR$ for current.

$$I = \frac{V}{R}$$

- Apply this form of Ohm's law to the current through each individual resistor and for the battery.

$$I_S = \frac{V_S}{R_{eq}} \quad I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_2}{R_2}$$
$$I_3 = \frac{V_3}{R_3} \quad I_N = \frac{V_N}{R_N}$$

- Replace the currents in the first equation with the expressions above.

$$\frac{V_S}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \cdots + \frac{V_N}{R_N}$$

- Write the relationship for the potential differences across resistors and the battery, when all are connected in parallel.

$$V_S = V_1 = V_2 = V_3 = \cdots = V_N$$

- Replace the potential differences in the equation above with V_S.

$$\frac{V_S}{R_{eq}} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} + \cdots + \frac{V_S}{R_N}$$

- Divide both sides of the equation by V_S.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

RESISTORS IN PARALLEL

For resistors connected in parallel, the inverse of the equivalent resistance is the sum of the inverses of the individual resistances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Quantity	Symbol	SI unit
equivalent resistance	R_{eq}	Ω (ohm)
resistance of the individual loads	$R_1, R_2, R_3, \ldots, R_N$	Ω (ohm)

MODEL PROBLEM

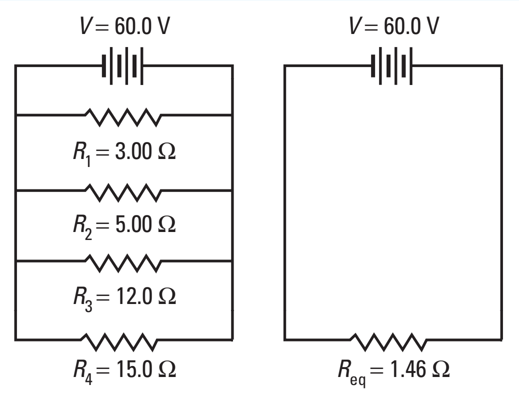
Resistors in Parallel

A 60 V battery is connected to four loads of 3.0 Ω, 5.0 Ω, 12.0 Ω, and 15.0 Ω in parallel.

(a) Find the equivalent resistance of the four combined loads.

(b) Find the total current leaving the battery.

(c) Find the current through the 12.0 Ω load.



Frame the Problem

- The four loads are connected *in parallel*; therefore, the *potential difference* across each load is the same as the *potential difference* provided by the *battery*.
- The *potential difference* across the battery and the *current* entering and leaving the battery would be *unchanged* if the four loads were replaced with one load having the *equivalent resistance*.

- After the current leaves the battery, it reaches branch points where it separates, and part of the current runs through each load.
- *Ohm's law* applies to each *individual load* or to the *combined load*. However, you must ensure that the current and potential difference are correct for the specific resistance that you use in the calculation.

Identify the Goal

The equivalent resistance, R_{eq}, of the four loads
The current, I_S, leaving the battery (source)
The current, I_3, through the 12.0 Ω load

PROBLEM TIPS

You might be surprised to find that the equivalent resistance of a parallel circuit is smaller than any of the individual resistances. However, when you think of each additional resistor in parallel as creating a greater cross sectional area for current to pass, it becomes reasonable.

Variables and Constants

Known	Unknown
$R_1 = 3.00\ \Omega$	R_{eq}
$R_2 = 5.00\ \Omega$	I_S
$R_3 = 12.0\ \Omega$	I_3
$R_4 = 15.0\ \Omega$	
$V_S = 60.0\ \text{V}$	

Strategy

Calculations

Use the equation for resistors in parallel and apply it to the four loads.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Substitute values for resistance and add.

$$\frac{1}{R_{eq}} = \frac{1}{3.00\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{12.0\ \Omega} + \frac{1}{15.0\ \Omega}$$

Find a common denominator.
Add.

$$\frac{1}{R_{eq}} = \frac{20}{60.0\ \Omega} + \frac{12}{60.0\ \Omega} + \frac{5}{60.0\ \Omega} + \frac{4}{60.0\ \Omega}$$

$$\frac{1}{R_{eq}} = \frac{41}{60.0\ \Omega}$$

Invert both sides of the equation. (If you invert an equality, it remains an equality.)

$$R_{eq} = \frac{60.0\ \Omega}{41}$$

Divide.

$$R_{eq} = 1.46\ \Omega$$

(a) The equivalent resistance of the four loads in parallel is 1.46 Ω.

Write Ohm's law, and solve for current.
Substitute numerical values.

$$V = IR$$

$$I_S = \frac{V_S}{R_{eq}}$$

$$I_S = \frac{60.0\ \text{V}}{1.46\ \Omega}$$

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Strategy

$1\,\frac{\text{V}}{\Omega}$ is equivalent to an A.

Calculations

$$I_S = 41.0\frac{\text{V}}{\Omega}$$

$$I_S = 41.0\text{ A}$$

(b) The current entering and leaving the battery is 41.0 A.

Write Ohm's law and solve for current, to find the current through the 12.0 Ω load.

$$V = IR$$

$$I_3 = \frac{60.0\text{ V}}{12.0\ \Omega}$$

$$I_3 = \frac{V_S}{R_3}$$

$$I_3 = 5.00\text{ A}$$

(c) Of the 41.0 A leaving the battery, 5.00 A are diverted to the 12.0 Ω load.

Validate

If you do a similar calculation for the current in each of the loads, the total current through all of the loads should equal 41 A.

$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2} \qquad I_4 = \frac{V_4}{R_4}$$

$$I_1 = \frac{60.0\text{ V}}{3.00\ \Omega} \qquad I_2 = \frac{60.0\text{ V}}{5.00\ \Omega} \qquad I_4 = \frac{60.0\text{ V}}{15.0\ \Omega}$$

$$I_1 = 20.0\text{ A} \qquad I_2 = 12.0\text{ A} \qquad I_4 = 4.00\text{ A}$$

$$I_{total} = 20.0\text{ A} + 12.0\text{ A} + 5.00\text{ A} + 4.00\text{ A}$$
$$I_{total} = 41.0\text{ A}$$

The answer is validated.

PROBLEM TIP

Using the *inverse key* [1/x] on your calculator makes these calculations easy. For example, to solve the above problem, enter the following sequence [3] [1/x] [+] [5] [1/x] [+] [1] [2] [1/x] [+] [1] [5] [1/x] [=]

The total on your calculator should now read 0.683… This is the sum of the inverses of the resistances and is the inverse of the expected answer, so now press [1/x]. The reading, 1.46…, becomes the value for the equivalent resistance. CAUTION: When using your calculator, it is very easy to forget the last step. Be sure to always press the inverse key after obtaining a sum.

PRACTICE PROBLEMS

Draw a circuit diagram for each problem below. As an aid, write the known values on the diagram.

32. A 9.00 V battery is supplying power to three light bulbs connected in parallel to each other. The resistances, R_1, R_2, and R_3, of the bulbs are 13.5 Ω, 9.00 Ω, and 6.75 Ω, respectively. Find the current through each load and the equivalent resistance of the circuit.

33. A light bulb and a heating coil are connected in parallel to a 45.0 V battery. The current from the battery is 9.75 A, of which 7.50 A passes through the heating coil. Find the resistances of the light bulb and the heating coil, and the equivalent resistance for the circuit.

34. A circuit contains a 12.0 Ω load in parallel with an unknown load. The current in the 12.0 Ω load is 3.20 A, while the current in the unknown load is 4.80 A. Find the resistance of the unknown load and the equivalent resistance for the two parallel loads.

35. A current of 4.80 A leaves a battery and separates into three currents running through three parallel loads. The current to the first load is 2.50 A, the current through the second load is 1.80 A, and the resistance of the third load is 108 Ω. Calculate (a) the equivalent resistance for the circuit, and (b) the resistance of the first and second loads.

Complex Circuits

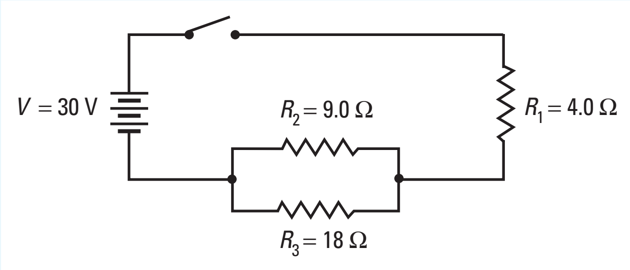
Many practical circuits consist of loads in a combination of parallel and series connections. To determine the characteristics of the circuit, you must analyze the circuit and recognize the way that different loads are connected in relation to each other. When a circuit branches, the loads in each branch must be grouped and treated as a single, or equivalent, load before they can be used in a calculation with other loads.

Figure 15.22 The circuitry shown here, typical of electronic equipment, illustrates the high level of complexity in circuitry of the household devices that we use every day.

MODEL PROBLEM

Resistors in Parallel

Find the equivalent resistance of the entire circuit shown in the diagram, as well as the current through, and the potential difference across, each load.



Frame the Problem

- The circuit consists of a *battery* and *three loads*. The battery generates a specific *potential difference* across the poles.
- The *current* driven by the potential difference of the battery depends on the *equivalent resistance* of the entire circuit.
- The circuit has two groups of resistors. Resistors R_2 and R_3 are in parallel with each other. Load R_1 is in series with the R_2–R_3 group.
- Define the R_2–R_3 group as Group A, and sketch the circuit with the equivalent load, R_A.
- Now, define the series group consisting of R_A and R_1 as Group B. Sketch the circuit with the equivalent load, R_B.
- Load R_B is the equivalent resistance of the entire circuit.

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Identify the Goal

The equivalent resistance, R_{eq}, of the circuit
The currents, I_1, I_2, and I_3, through the loads
The potential differences, V_1, V_2, and V_3, across the loads

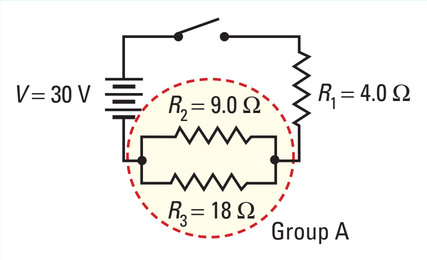
Variables and Constants

Known	Unknown	
$R_1 = 4.0\ \Omega$	R_{eq}	V_1
$R_2 = 9.0\ \Omega$	I_1	V_2
$R_3 = 18\ \Omega$	I_2	V_3
$V_S = 30$ V	I_3	

Strategy / Calculations

Find the equivalent resistance for the parallel Group A resistors.

$$\frac{1}{R_A} = \frac{1}{R_2} + \frac{1}{R_3}$$



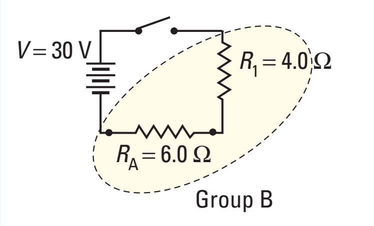
Find a common denominator.

$$\frac{1}{R_A} = \frac{1}{9.0\ \Omega} + \frac{1}{18\ \Omega}$$

$$\frac{1}{R_A} = \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega}$$

Simplify.

$$\frac{1}{R_A} = \frac{3}{18\ \Omega}$$



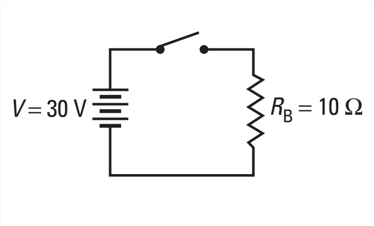
Invert both sides of the equation.

$$\frac{1}{R_A} = \frac{1}{6.0\ \Omega}$$

$$R_A = 6.0\ \Omega$$

Find the equivalent resistance of the series Group B.

$$R_B = R_A + R_1$$
$$R_B = 4.0\ \Omega + 6.0\ \Omega$$
$$R_B = 10\ \Omega$$



Since there is now one equivalent resistor in the circuit, the effective resistance of the entire circuit is $R_{eq} = 10\ \Omega$.

Find the current entering and leaving the battery, using the potential difference of the battery and the total effective resistance of the circuit.

$$V = IR$$

$$I_S = \frac{V_S}{R_{eq}}$$

$$I_S = \frac{30\ \text{V}}{10\ \Omega}$$

$$I_S = 3.0\ \text{A}$$

Since there are no branches in the circuit between the battery and load R_1, all of the current leaving the battery passes through R_1. Therefore, $I_1 = 3.0$ A.

Knowing the current through R_1, and its resistance, you can use Ohm's law to find V_1.

$$V_1 = I_1R_1$$
$$V_1 = (3.0\ \text{A})(4.0\ \Omega)$$
$$V_1 = 12\ \text{V}$$

The potential difference across load 1 is 12 V.

The loads R_1 and R_A form a complete path from the anode to the cathode of the battery; therefore, the sum of the potential drops across these loads must equal that of the battery.

$$V_S = V_1 + V_A$$
$$30\ \text{V} = 12\ \text{V} + V_A$$
$$30\ \text{V} - 12\ \text{V} = V_A$$
$$V_A = 18\ \text{V}$$

Since the potential difference across a parallel connection is the same for both pathways, $V_2 = 18$ V and $V_3 = 18$ V.

Knowing the potential difference across R_2 and R_3, you can find the current through each load by using Ohm's law.

$$V = IR \qquad V = IR$$
$$I_2 = \frac{V_2}{R_2} \qquad I_3 = \frac{V_2}{R_3}$$
$$I_2 = \frac{18\ \text{V}}{9.0\ \Omega} \qquad I_3 = \frac{18\ \text{V}}{18\ \Omega}$$
$$I_2 = 2.0\ \text{A} \qquad I_3 = 1.0\ \text{A}$$

The current through load 2 is 2.0 A, and the current through load 3 is 1.0 A.

To summarize, the 30 V battery causes a current of 3.0 A to move through the circuit. All of the current passes through the 4.0 Ω load, but then splits into two parts, with 2.0 A going through the 9.0 Ω load and 1.0 A going through the 18 Ω load. The potential drops across the 4.0 Ω, 9.0 Ω, and 18 Ω loads are 12 V, 18 V, and 18 V, respectively.

PROBLEM TIP

When working with complex circuits, look for the smallest groups of loads that are connected *only* in parallel or *only* in series. Find the equivalent resistance of the group, then redraw the circuit with one load representing the equivalent resistance of the group. Begin again.

Validate

The current to Group A, known to be 3.0 A, was split into two parts. The portion of the current in each of the branches of Group A was inversely proportional to the resistance in the branch. The larger 18 Ω load allowed half the current that the smaller 9.0 Ω load allowed. The sum of the currents through R_2 and R_3 is equal to the current through R_1, which is expected to be true.

The potential difference across Group A should be the same as that of one load with a resistance, R_A, with the total current passing through it. Check this potential difference and compare it with the 18 V found by subtracting 12 V across load 1 from the total of 30 V.

$$V_A = I_SR_A = (3.0\ \text{A})(6.0\ \Omega) = 18\ \text{V}$$

The values are in agreement. The answers are consistent.

continued ►

continued from previous page

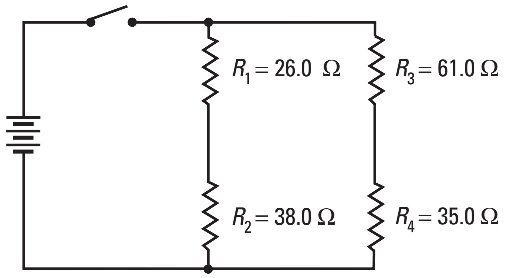
PRACTICE PROBLEMS

36. For the circuit in the diagram shown below, the potential difference of the power supply is 144 V. Calculate

(a) the equivalent resistance for the circuit

(b) the current through R_1

(c) the potential difference across R_3

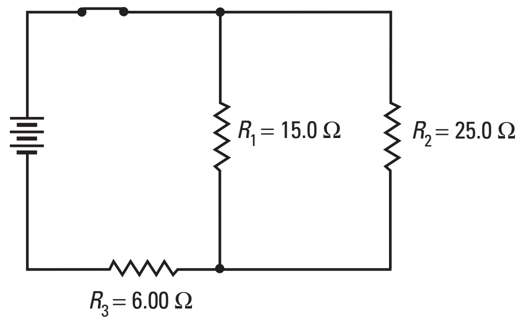


37. For the circuit shown in the diagram below, the potential difference of the power supply is 25.0 V. Calculate

(a) the equivalent resistance of the circuit

(b) the potential difference across R_3

(c) the current through R_1



Internal Resistance

The objective of Part 2 of Investigation 15-B (Current, Resistance, and Potential Difference) was to find out how the current varied with resistance. Each time you changed the resistance, you had to reset the potential difference across the load to the desired value. The circuit behaved as if some phantom resistance was affecting the potential difference. To an extent, that is exactly what was happening — the phantom resistance was the internal resistance of the battery or power supply.

Figure 15.23 A chemical reaction separates electric charge creating a potential difference between the terminals. Internal resistance within the battery reduces the amount of voltage available to an external circuit.

To develop an understanding of **internal resistance**, consider a gasoline engine being used to power a ski lift. Gasoline engines convert energy from the fuel into mechanical energy, which is then used, in part, to pull the skiers up the hill. No matter how efficient the motor is, however, it must always use some of the energy to overcome the friction inside the motor itself. In fact, as the number of skiers on the lift increases, the amount of energy the motor uses to run itself also increases.

The process involved in an electric circuit is very similar to the motor driving the ski lift. Inside the battery, chemical reactions create a potential difference, called the **electromotive force** (*emf,* represented in equations by $\mathscr{E}$). If there was no internal resistance inside the battery, the potential difference across its anode and cathode (sometimes called the **terminal voltage**) would be exactly equal to the *emf*. However, when a battery is connected to a circuit and current is flowing, some of the *emf* must be used to cause the current to flow through the internal resistance (r) of the battery itself. Therefore, the terminal voltage (V_S) of the battery is always less than the *emf* by an amount equal to the potential difference across the internal resistance ($V_{int} = I \cdot r$). You can find the terminal voltage by using the equation in the following box.

TERMINAL VOLTAGE AND *emf*

The terminal voltage (or potential difference across the poles) of a battery is the difference of the *emf* ($\mathscr{E}$) of the battery and the potential drop across the internal resistance of the battery.

$$V_S = \mathscr{E} - V_{int}$$

Quantity	Symbol	SI unit
terminal voltage	V_S	V (volt)
electromotive force	$\mathscr{E}$	V (volt)
internal potential drop of a battery	V_{int}	V (volt)

PHYSICS FILE

The electromotive force of a battery is not a force at all. It is a potential difference. However, the term "electromotive force" came into common usage when electric phenomena were not as well understood as they are today. Nevertheless, the term "electromotive force" is still used by physicists.

If no current is passing through a battery, then the potential difference across the internal resistance will be zero ($V_{int} = 0$). As a result, the potential difference across its terminals (V_S) will be equal to the *emf* ($\mathscr{E}$).

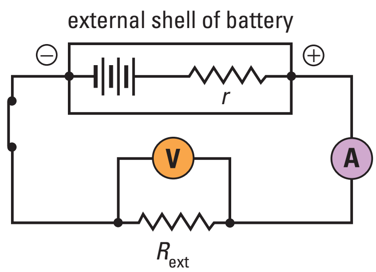


Figure 15.24 A battery has an internal resistance (r) that is in series with the *emf* ($\mathscr{E}$) of the cell.

MODEL PROBLEM

Terminal Voltage versus *emf* of a Battery

A battery with an *emf* of 9.00 V has an internal resistance of 0.0500 Ω. Calculate the potential difference lost to the internal resistance, and the terminal voltage of the battery, if it is connected to an external resistance of 4.00 Ω.

Frame the Problem

- A battery is connected to a closed circuit; thus, a *current* is flowing.
- Due to the *internal resistance* of the battery and the current, a *potential drop* occurs inside the battery.
- The *potential drop* across the internal resistance depends on the amount of *current* flowing in the circuit.
- The *terminal voltage* is lower than the *emf* due to the loss of energy to the internal resistance.

Identify the Goal

The potential difference (V_{int}) lost by current passing through the internal resistance of the battery

The terminal voltage (V_S), or potential difference across the poles, of the battery

Variables and Constants

Known	Unknown
$\mathscr{E} = 9.00\ \text{V}$	V_S
$r = 0.0500\ \Omega$	V_{int}
$R_{ext} = 4.00\ \Omega$	I_S
	$\mathscr{E}_s$

Strategy

To find the current flowing in the circuit, you need to know the equivalent resistance of the circuit. Since the internal resistance is in series with the external resistance, use the equation for series circuits.

Calculations

$$R_{emf} = r + R_{eq}$$

$$R_{emf} = 0.0500\ \Omega + 4.00\ \Omega$$

$$R_{emf} = 4.05\ \Omega$$

Strategy

Use the *emf* ($\mathscr{E}$), and resistance to the *emf* (R_{emf}) in Ohm's law to find the current in the circuit.

Calculations

$V = IR$

$I_S = \frac{\mathscr{E}}{R_{emf}}$

$I_S = \frac{9.00\ V}{4.05\ V}$

$I_S = 2.22\ A$

Find the internal potential drop of the battery by using Ohm's law, the current, and the internal resistance.

$V_{int} = I_S r$

$V_{int} = (2.22\ A)(0.0500\ \Omega)$

$V_{int} = 0.111\ V$

Find the terminal voltage from the *emf* and the potential drop due to the internal resistance.

$V_S = \mathscr{E} - V_{int}$

$= 9.00\ V - 0.111\ V$

$= 8.89\ V$

The potential difference across the internal resistance is 0.111 V, causing the *emf* of the battery to be reduced to the terminal voltage of 8.89 V. This is the portion of the *emf* available to the external circuit.

Validate

In order for a battery to be useful, you would expect that the loss of potential difference due to its internal resistance would be very small, compared to the terminal voltage. In this case, the loss (0.111 V) is just over 1 percent of the terminal voltage (8.898 V). The answer is reasonable.

PRACTICE PROBLEMS

38. A battery has an *emf* of 15.0 V and an internal resistance of 0.0800 Ω.

(a) What is the terminal voltage if the current to the circuit is 2.50 A?

(b) What is the terminal voltage when the current increases to 5.00 A?

39. A battery has an internal resistance of 0.120 Ω. The terminal voltage of the battery is 10.6 V when a current of 7.00 A flows from it.

(a) What is its *emf*?

(b) What would be the potential difference of its terminals if the current was 2.20 A?

INVESTIGATION 15-C

Internal Resistance of a Dry Cell

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

You can usually measure the resistance of a load by connecting a voltmeter across the load and connecting an ammeter in series with the load. However, you cannot attach a voltmeter across the internal resistance of a battery; you can attach a voltmeter only across the poles of a battery. How, then, can you measure the internal resistance of a dry cell?

Problem

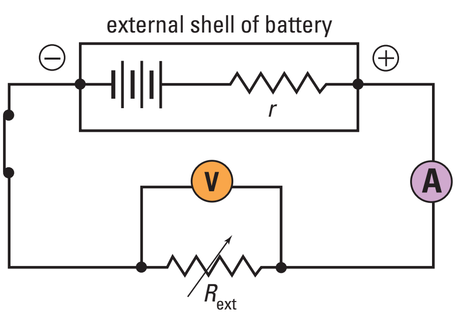
Determine the internal resistance of a dry cell.

Equipment

- $1\frac{1}{2}$ volt D-cell (or 6 V battery)
- variable resistor
- voltmeter
- ammeter
- conductors with alligator clips

Procedure

1. Measure and record the electromotive force (*emf,* represented by $\mathcal{E}$) of the battery. This is the potential difference of the battery before it is connected to the circuit. Record all data to at least three significant digits.
2. Make a data table with the column headings: Trial, Terminal Voltage (V_S), Current (I_S), Equivalent Resistance (R_{eq}), Resistance to the *emf* (R_{emf}), Internal Resistance (r).
3. Connect the apparatus as shown in the diagram.



CAUTION Inspect your connections carefully. Refer to the wiring instructions given in Investigation 15-B on page 709 if you need to refresh your memory.

CAUTION Open the switch or disconnect the circuit when you are not taking readings.

4. Adjust the variable resistor so that the current is about 1.0 A. Then, record the readings for the voltmeter (V_S) and ammeter (I_S).
5. Reduce the resistance of the load and record the voltmeter and ammeter readings.
6. Obtain at least four sets of data readings by reducing the resistance after each trial.
7. After taking the last reading from the circuit, remove the cell from the circuit and measure the *emf* of the cell again to confirm that it has not been diminished significantly.

Analyze and Conclude

1. For each trial, calculate and enter into your data table the value of the equivalent resistance, R_{eq}.
$$\left(R_{eq} = \frac{V_S}{I_S}\right)$$
2. For each trial, calculate and enter into your data table the value of the total resistance, R_{emf}.
$$\left(R_{emf} = \frac{\mathcal{E}}{I_S}\right)$$
3. For each trial, calculate and record in your data table the value of the resistance offered to the *emf* ($r = R_{emf} - R_{eq}$).
4. Explain why the terminal voltage decreases when the current increases.
5. Is the calculated value for the internal resistance constant for all trials? Find the average and the percent error for your results.

15.4 Section Review

1. K/U Classify each of the following statements as characteristics of either a series or a parallel circuit.
   (a) The potential difference across the power supply is the same as the potential difference across each of the circuit elements.
   (b) The current through the power supply is the same as the current through each of the circuit elements.
   (c) The current through the power supply is the same as the sum of the currents through each of the circuit elements.
   (d) The equivalent resistance is the sum of the resistance of all of the resistors.
   (e) The potential difference across the power supply is the same as the sum of the potential differences across all of the circuit elements.
   (f) The current is the same at every point in the circuit.
   (g) The reciprocal of the equivalent resistance is the sum of the reciprocals of each resistance in the circuit.
   (h) The current leaving the branches of the power supply goes through different circuit elements and then combines before returning to the power supply.
2. K/U A series circuit has four resistors, A, B, C, and D. Describe, in detail, the steps you would take in order to find the potential difference across resistor B.
3. K/U A parallel circuit has three resistors, A, B, and C. Describe, in detail, the steps you would take to find the current through resistor C. Describe a completely different method for finding the same value.
4. C A complex circuit has three resistors, A, B, and C. The equivalent resistance of the circuit is greater than the resistance of A, but less than the resistance of either B or C. Sketch a circuit in which this would be possible. Explain why.
5. K/U Under what conditions would the equivalent resistance of a circuit be smaller than the resistance of any one of the loads in the circuit? Explain.
6. I In Investigation 15-C (Internal Resistance of a Dry Cell), why were you cautioned to open the switch when you were not taking readings?
7. I Study the circuit below, in which a battery is connected to four resistors in parallel. Initially, none of the switches is closed. Assume that you close the switches one at a time and record the reading on the voltmeter after closing each switch. Describe what would happen to the voltmeter reading as you close successive switches. Explain why this would happen.